Visible Trajectory of LEO Satellite Networks

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***Abstract*—This paper proposes a new systematic analysis framework for the LEO satellite networks by focusing on the orbit geometry. By modeling LEO satellite orbits as a Poisson line process, we characterize the distribution of the total length of the visible orbit region observed by the typical user.**

***Index Terms*—LEO satellites, Stochastic geometry**

I. Introduction

The satellite networks have been widely considered as the solution to support the global coverage in the beyond 5G, or 6G era [1]. Especially since low-Earth-orbit (LEO) satellites provide low latency and high data rates, understanding the LEO satellite networks is significant in terms of system parameters to optimize satellite deployment.

In order to analyze the network performance, stochastic geometry has been widely used to analyze the network performances by spatially averaging the performances of wireless networks. Prior works have been conducted on this mathematical tool to analyze the LEO satellite networks [2], but the orbit geometry has not been much considered. In this paper, we focus on the length of the visible region of orbits by adopting stochastic geometry.

II. System Model

This section presents the network model for LEO satellite networks, highlighting the orbit geometry.

We assume that the Earth is a perfect sphere with radius r_{E} and each orbit is a circle with radius r where orbits and the Earth share the same center. r_{h} is the altitude of the LEO satellites where $r_{\mathsf{h}} = r - r_{\mathsf{E}}$.

Since each orbit is on the plane which is a two-dimensional subspace of the three-dimensional Euclidean space, $\mathbb{R}^3$, it is completely characterized by a normal unit-vector of the plane. So, we can make a one-to-one correspondence between a satellite orbit and the normal unit-vector with respect to the plane containing that orbit as in Figure 1. Let p_k be the k-th orbit and $\mathbf{v}_k$ be the unit-normal vector which is associated with p_k. Let θ_k and ϕ_k be the polar angle and azimuth angle of $\mathbf{v}_k$, i.e., $\mathbf{v}_k = (1, \theta_k, \phi_k)$ in the spherical coordinate system where $\theta_k \in [0, \pi]$ and $\phi_k \in [0, 2\pi)$. So, we can map an orbit p_k to a point $\mathbf{x}_k = (\theta_k, \phi_k)$ in (θ, ϕ) domain as in Figure 1. We will distribute the set of points $\{\mathbf{x}\} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$ on the (θ, ϕ) domain according to a homogeneous Poisson point process (PPP) with an intensity λ. In other words, N is a Poisson random variable with mean $2\pi^2\lambda$.

We will consider a user's observable satellite trajectory and place the user at $(0, 0, r_{\mathsf{E}})$ in the Cartesian coordinate system without loss of generality. We refer to this user as the typical user. By assuming users can observe satellites above the minimum elevation angle ω, we denote the visible region on the spherical surface with radius r observed by the typical user by R where

$$R = \{(x, y, z) \in \mathbb{R}^3 : \{x^2 + y^2 + z^2 = r^2\} \cap \{z > d \sin\omega + r_{\mathsf{E}}\}\}, \quad (1)$$

and

$$d = -r_{\mathsf{E}} \sin\omega + \sqrt{(r_{\mathsf{E}} \sin\omega)^2 + 2 r_{\mathsf{E}} r_{\mathsf{h}} + r_{\mathsf{h}}^2} \quad (2)$$

which is the maximum distance between the user at $(0, 0, r_{\mathsf{E}})$ and R. Equation (2) is obtained by the cosine law.

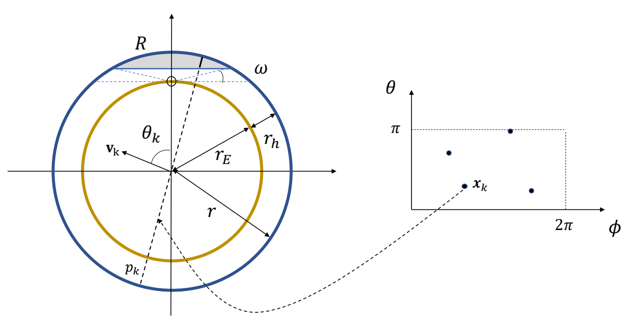

Fig. 1. Illustration of network geometry.

III. Visible Trajectory

In this section, we will analyze the total length of visible orbits by the user placed at $(0, 0, r_{\mathsf{E}})$.

Let l_k be length of the orbit p_k passing R, i.e., $l_k = |p_k \cap R|$. By definition, l_k is invariant over the azimuth angle ϕ_k.

Lemma 1: The length of orbit p_k passing the visible region R is

$$l_k = \begin{cases} l(\theta_k) & \text{for } |\theta - \frac{\pi}{2}| \leq \arccos\left(\frac{r_b}{R}\right) \\ 0 & \text{otherwise} \end{cases}, \quad (3)$$

where

$$l(\theta) = r \arccos\left(\frac{2r_b^2}{r^2 \sin^2\theta} - 1\right) \quad (4)$$

and $r_b = d\sin\omega + r_{\mathsf{E}}$.

The k-th orbit r_k does not pass the visible region R if θ_k is not in $(\frac{\pi}{2} - \arccos\left(\frac{r_b}{R}\right), \frac{\pi}{2} + \arccos\left(\frac{r_b}{R}\right))$. When r_k passes R, l_k is obtained by the multiplication of the radius r and the angle of the arc, $r_k \cap R$.

Since $\arccos$ function is a decreasing function, l_k is maximized when the argument is minimized. So, the maximum l_k is obtained as $r\arccos\left(\frac{2r_b^2}{r^2} - 1\right)$ when θ_k is $\frac{\pi}{2}$. This result confirms the intuition that when an orbit passes the zenith of the user, he can observe the maximum length of that orbit. Also, we can check that l_k is independent of the azimuth angle ϕ_k.

Now, we will analyze the total length of the observable trajectory based on Lemma 1. Let l be the total length of visible orbit which is given by

$$l = \sum_{k=1}^{N} l_k \quad (5)$$

where N is the number of orbits.

Theorem 1: The Laplace transform of l is

$$\mathcal{L}_l(s) = \exp\left(-2\pi\lambda \int_0^{\pi} \left(1 - e^{-sl(\theta)}\right) d\theta\right). \quad (6)$$

The proof comes from the probability generating functional (PGFL) of PPP [3].

The Laplace transform of a random variable is connected to the moments of that random variable. By leveraging the relation

$$\mathbb{E}[X^n] = (-1)^n \frac{d^n}{ds^n}\mathcal{L}_X(s)\bigg|_{s=0}, \quad (7)$$

we can obtain the following corollary.

Corollary 1: The mean of l is

$$\mathbb{E}[l] = 2\pi\lambda \int_0^{\pi} l(\theta)d\theta, \quad (8)$$

which is obtained by applying (7) to the result of Theorem 1.

IV. Numerical Experiments

Figure 2 illustrates the mean of l obtained in Corollary 1. In this figure, we set $R_{\mathsf{E}} = 6371(km)$ and $R_{\mathsf{h}} = 500(km)$. From this figure, we can observe that the mean of l is proportional to the orbit density λ. Also, the mean of l decreases as the minimum elevation angle ω increases since the area of visible surface R is reduced by increasing ω. We can check that (8) is well matched to the simulation results.

Since LEO satellites are located on orbits, the visibility of satellites highly depends on l. When the visibility is not secured enough, the typical user will not be served by any satellite. By investigating the behavior of the visible orbit lengths, it can be extended to the coverage analysis of LEO satellite networks.

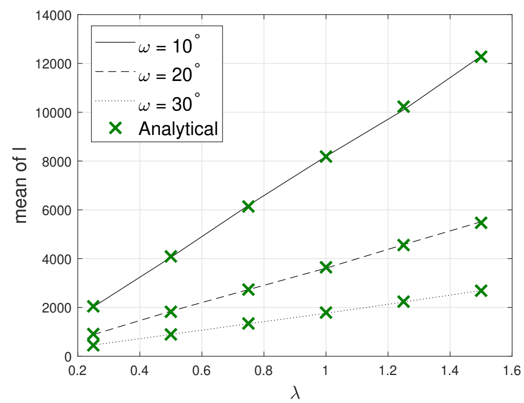

Fig. 2. Illustrations of the mean of l under $R_{\mathsf{E}} = 6371(km)$, $R_{\mathsf{h}} = 500(km)$.

V. Conclusion

This paper analyzed the visible trajectory observed by the typical user under LEO satellite networks, highlighting the orbit geometry. By modeling the orbit as a Poisson line process, we obtain the distribution of the sum of visible orbit regions. With numerical experiments, we verified that the analytical expressions are well-matched with the simulation experiments and provide intuitions on how network parameters are connected to the visible regions.

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