Constructions of Optical MIMO Priority Queues With Time-Varying Service Capacity

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***Abstract*—One of the main challenges in all-optical packet switching is to design optical buffers for packet conflict resolution. In this paper, we consider a very general type of buffering schemes, namely, optical N-to-K priority queues with time-varying service capacity, where each packet is associated with a unique priority upon its arrival, at time slot t at most $c(t)$ highest-priority packets are sent out from the queue if there are packets in the queue and the service capacity $c(t)$ $(0 \leq c(t) \leq K)$ of the queue is not zero, and up to N lowest-priority packets are dumped from the queue if there is a buffer overflow. We extend and generalize our previous constructions [8] of optical priority queues under a priority-based routing policy from single input/output to multiple inputs/outputs. The main contributions of this paper are as follows: (i) The priority queues considered in this paper subsume those considered in all previous works as special cases. (ii) Our queueing model with time-varying service capacity is not only more general but also more realistic than that with fixed service capacity previously studied in the literature. (iii) For the special case that $N = K = 1$, our constructions in this paper subsume those in [8] as special cases. (iv) For the special case that $N = K$, we show that an optical N-to-N priority queue with buffer size $2^{O(\sqrt{M/N})}$ (exponential in $\sqrt{M/N}$) can be constructed by using an optical $(M+2N)\times(M+2N)$ (bufferless) crossbar switch and M fiber delay lines, which substantially improves the best result $O(M^3/N^2)$ (polynomial in M/N) in the literature.**

***Index Terms*—MIMO, optical buffers, optical queues, optical switches, priority queues, time-varying service capacity.**

I. Introduction

An important and challenging issue in all-optical packet switching is the design of optical buffers for contention resolution among packets competing for the same resources so as to eliminate the notorious optical-electrical-optical (O-E-O) problem and keep up with the ever-growing pace of optical fiber link capacity. As optical random-access memory (RAM) is not available yet, one of the approaches currently available is to use optical fiber delay lines to store optical packets and use optical (bufferless) crossbar switches to route optical packets through the fiber delay lines in a carefully designed manner so that packets can be routed to the right places at the right times, and hence exact emulations of the desired optical buffers can be achieved.

The primary issue in such a Switched-Delay-Line (SDL) approach is as follows: (i) The design of the *delays* of the optical fiber delay lines. (ii) The design of the *routing policy* performed by the optical crossbar switches, namely, the design of how the inputs are connected to the outputs of the optical crossbar switches. Apparently, such an optical buffer does not have random-access capability and is not flexible to use. This makes SDL design of optical buffers very complicated and difficult.

In many packet-switched networks, the buffering schemes needed have certain special arrival/departure patterns. By exploiting the special arrival/departure patterns of such buffering schemes, the SDL approach has been successfully used to construct a variety of optical queues in the last two-plus decades. including, in particular, the constructions of optical priority queues in [1]–[9]. Although our focus on the constructions of optical priority queues in this paper is from the theoretical point of view, we are aware of the importance of certain practical feasibility issues such as router buffer sizing problem, fault-tolerant capability, and limitation on the number of times that an optical packet can recirculate through the optical switches and the fiber delay lines (see Sections V-A and V-C in [8] for details). For review articles on SDL constructions of optical queues as well as related implementation and feasibility issues, we refer to [10]–[15] and the references therein.

In this paper, we focus on SDL constructions of optical priority queues with multiple inputs/outputs and time-varying service capacity. Among the existing works [1]–[9] on the constructions of optical priority queues, the works [1]–[8] are on optical priority queues with single input and single output and the work [9] is the only one on optical priority queues with multiple inputs and multiple outputs. It was first proposed in [1] to use an optical $(M + 2) \times (M + 2)$ (bufferless) crossbar switch and M fiber delay lines for the construction of an optical priority queue and the buffer size achieved is $O(M^2)$. The proof in [1] was made simpler in [2]. The buffer size $O(M^2)$ achieved in [1] was improved to $O(M^3)$ in [3], was improved to $O(M^c)$ for any positive integer c in [6], and was substantially improved to $2^{O(\sqrt{M})}$ in [8]. By extending the constructions in [3] from single input/output to multiple inputs/outputs, it was shown in [9] that an optical N-to-K priority queue can be constructed using an optical $(M+N+K)\times(M+N+K)$ (bufferless) crossbar switch and M fiber delay lines. For the special case that $N = K$, tt was also shown in [9] that the buffer size that can be achieved is $O(M^3/N^2)$. Furthermore, a theoretical upper bound $(K^2 + 2K + N)2^{(M-N-K)\log_2(1+1/N)} = 2^{O(M/N)}$ on the buffer size that can be achieved by using optical (bufferless) crossbar switches and M fiber delay lines was given in [9] (we note that this upper bound reduces to the upper bound 2^M in [1] when $N = K = 1$).

An N-to-K priority queue with time-varying service capacity $c(t)$ and buffer size B is a network element with N arrival links, K departure links, and N loss links (see Figure 1). The

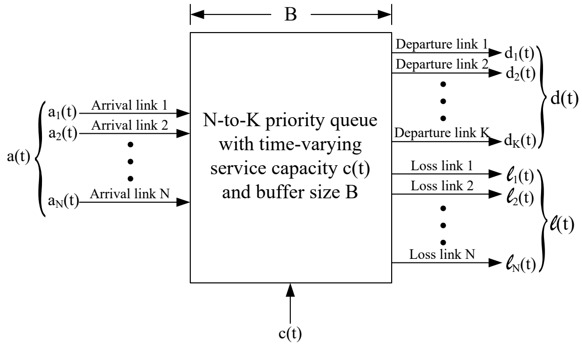

Fig. 1. An N-to-K priority queue with time-varying service capacity $c(t)$ and buffer size B.

service capacity of the queue at time slot t is $c(t)$, and it means that the queue can send out at most $c(t)$ packets from the departure links at time slot t, where $0 \leq c(t) \leq K$ (as there are only K departure links). Each packet is associated with a unique priority upon its arrival so that the following two properties are satisfied: (i) *Total order*: every packet in the queue has a distinct priority. (ii) *Relative order*: the relative priority order between any two packets remains unchanged as long as they are in the queue. When there are packets in the queue and the service capacity of the queue is not zero at time slot t, at most $c(t)$ packets with the *highest* priorities are sent out from the departure links. When there is a buffer overflow, up to N packets with the *lowest* priorities are dumped through the loss links.

It is clear that priority queues are very general types of queues since packet arrival times, service capacities, and priority assignment of packets can be arbitrary (as long as the total order property and the relative order property are satisfied). In particular, they subsume FIFO (resp., LIFO) queues as special cases, where packets with earlier (resp., later) arrival times have higher priorities. Furthermore, they can be used to implement optical output-buffered switches that support the packetized version of the generalized processor sharing (PGPS) policy (see Figure 3 in [9] for an illustration).

The constructions of optical N-to-K priority queues with time-varying service capacity $c(t)$ in this paper use the feedback system in Figure 2 consisting of an optical $(kmn+N+K)\times(kmn+N+K)$ (bufferless) crossbar switch and k groups of optical n-to-1 FIFO multiplexers with delay one (nFM1's), where the i^{th} group has m parallel optical nFM1's with the same buffer size B_i ($B_i \geq 1$) for $i = 1, 2, \ldots, k$. An optical nFM1 with buffer size B is defined as the concatenation of an optical n-to-1 FIFO multiplexer (nFM) with buffer size $B-1$ and a fiber delay line with delay equal to one, where the departure link of the nFM is connected to the input link of the fiber delay line with delay one and an nFM with buffer size $B-1$ is simply an n-to-1 priority queue with buffer size $B-1$ whose service capacity is always equal to one.

In this paper, we extend and generalize the constructions in [8] and show in Theorem 7 that the feedback system in Figure 2 can be operated as an optical N-to-K priority queue with

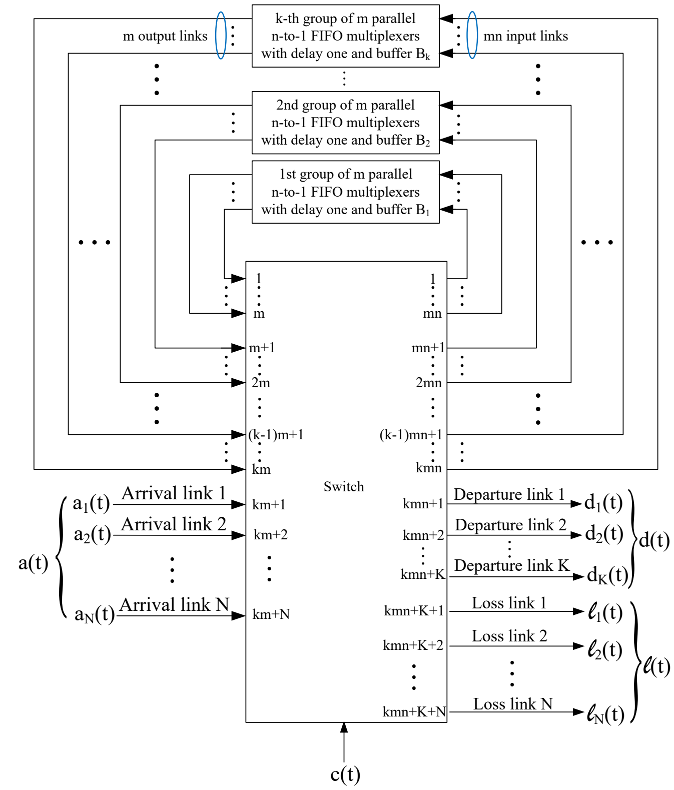

Fig. 2. A construction of an optical N-to-K priority queue with time-varying service capacity $c(t)$ by using an optical $(kmn+N+K)\times(kmn+N+K)$ (bufferless) crossbar switch and k groups of m parallel optical n-to-1 FIFO multiplexers with delay one.

time-varying service capacity $c(t)$. The main contributions of this paper are as follows: (i) The priority queues considered in this paper subsume those considered in all pervious works [1]–[9] on the constructions of optical priority queues as special cases (see Remark 1(i) for details). (ii) Our queueing model with time-varying service capacity is not only more general but also more realistic than that with fixed service capacity in the only work [9] on the constructions of optical priority queues with multiple inputs and multiple outputs (see Remark 1(ii) for details). (iii) For the special case that $N = K = 1$, our constructions in Theorem 7 subsume those in [9] as special cases (see Remark 8 for details). (iv) For the special case that $N = K$, we show that an optical N-to-N priority queue with buffer size $2^{O(\sqrt{M/N})}$ can be constructed by using an optical $(M+2N)\times(M+2N)$ (bufferless) crossbar switch and M fiber delay lines. Our result $2^{O(\sqrt{M/N})}$ (exponential in $\sqrt{M/N}$) greatly improves the best result $O(M^3/N^2)$ (polynomial in M/N) in the literature (see Section II-D for details).

The rest of this paper is organized as follows. First, we give a more detailed description of N-to-K priority queues with time-varying service capacity in Section II-A. Then we derive two *intrinsic* properties of buffering tags that are independent of how an N-to-K priority queue is implemented in Section II-B, and use these intrinsic properties to derive two basic properties of buffering tags for the constructions in

Figure 2 under our priority-based routing policy performed by the crossbar switch in Section II-C. Finally, we show in Section II-D that the feedback system in Figure 2 can be operated as an optical N-to-K priority queue under our priority-based routing policy, and perform a complexity analysis for our constructions with maximum buffer size. We conclude this paper in Section III.

II. N-to-K Priority Queues and Priority-Based Routing Policy

As in most works on SDL constructions of optical queues in the literature, in this paper we consider the following discrete-time settings: (i) Time is slotted and synchronized. (ii) Packets are of the same size so that a packet can be transmitted through a link within a time slot. (iii) An optical $M \times M$ (bufferless) crossbar switch is a network element with M input links and M output links that can realize all of the $M\,!$ permutations between its inputs and its outputs. (iv) A fiber delay line with delay d is a network element with one input link and one output link that requires d time slots for a packet to traverse through. We note that variable-size packets can be easily taken care of by introducing packet segmentation at the source and packet reassembly at the destination. For conciseness, we write time slot t as "slot t" in this paper.

Note that there is at most one packet in a link at any slot (as a packet can be transmitted through a link within a slot by assumption (ii) above). Thus, we can characterize a link by its link state, say a link is in state 1 (resp., 0) at slot t if there is a packet (resp., there is no packet) in the link at slot t.

Furthermore, in this paper we assume that every network element is started from an empty system at slot $t = 0$.

A. N-to-K Priority Queues

For an N-to-K priority queue as shown in Figure 1, we denote $a_i(t)$ (resp., $d_i(t)$ and $\ell_i(t)$) as the link state of arrival (resp., departure and loss) link i at slot t for $i = 1, 2, \ldots, N$ (resp., $i = 1, 2, \ldots, K$ and $i = 1, 2, \ldots, N$). Let $a(t) = \sum_{i=1}^{N} a_i(t)$ (resp., $d(t) = \sum_{i=1}^{K} d_i(t)$ and $\ell(t) = \sum_{i=1}^{N} \ell_i(t)$) be the number of arrival (resp., departure and loss) packets from the arrival (resp., departure and loss) links at slot t. Also let $q(t)$ be the number of packets stored in the buffer of the queue at slot t.

Then an N-to-K priority queue with time-varying service capacity $c(t)$ and buffer size B is characterized by the following five properties at all slots $t \geq 1$:

(P1) Flow conservation: Packets arriving from the arrival links are either stored in the buffer or transmitted through the departure links or the loss links. Therefore, we have $q(t) = q(t-1) + a(t) - d(t) - \ell(t)$.

(P2) Nonidling: If there are packets in the queue at slot t and the service capacity of the queue at slot t is not zero, i.e., $q(t-1)+a(t) > 0$ and $c(t) > 0$, then there are $\min\{q(t-1)+a(t), c(t)\}$ departure packets at slot t (as the queue can send out at most $c(t)$ packets from the departure links at slot t); otherwise, there are no departure packets at slot t. Therefore, we have $d(t) = \min\{q(t-1) + a(t), c(t)\}$.

(P3) Maximum buffer usage: If there is a buffer overflow at slot t, i.e., $q(t-1) + a(t) - d(t) > B$ (note that $q(t-1) + a(t) - d(t)$ is the number of packets in the queue, excluding the departure packets, at slot t), then there are $q(t-1)+a(t)-d(t) - B$ loss packets at slot t; otherwise, there are no loss packets at slot t. Therefore, we have $\ell(t) = (q(t-1)+a(t)-d(t) - B)^+$, where we have denoted $x^+ = \max\{x, 0\}$.

(P4) Priority departure with prioritized departure links: If there are departure packets at slot t, i.e., $d(t) > 0$, then the departure packets are the $d(t)$ highest-priority packets in the queue at slot t and they depart from departure links $1, 2, \ldots, d(t)$ in the order of decreasing priorities.

(P5) Priority loss with prioritized loss links: If there are loss packets at slot t, i.e., $\ell(t) > 0$, then the loss packets are the $\ell(t)$ lowest-priority packets in the queue at slot t and they are dumped through loss links $1, 2, \ldots, \ell(t)$ in the order of decreasing priorities.

Remark 1 *(i) The priority queues considered in this paper subsume those in [1]–[9] as special cases. Specifically, the N-to-K priority queue considered in this paper specializes to those in [1]–[8] when $N = K = 1$, and specializes to that in [9] when its service capacity $c(t)$ assumes only two values, i.e., 0 and K. We note that in [9] the notation $c(t)$ is used to indicate whether the queue is enabled or not at slot t, namely, if the queue is enabled (resp., disabled) at slot t, then $c(t) = 1$ (resp., $c(t) = 0$) and at most K packets (resp., no packets) can be sent out from the departure links at slot t, meaning that its service capacity is K (resp., 0) at slot t.*

(ii) Our queueing model with time-varying service capacity is not only more general but also more realistic than that in [9]. For example, consider the scenario that the K departure links are shared among many network elements. In such a scenario, the service capacity of the queue is the number of departure links allocated to the queue for its use by certain resource management scheme (which is more involved and is beyond the scope of this paper). Clearly, it is more flexible and more realistic for the resource management scheme to allocate any number of the departure links to the queue at any slot (note that the queue in [9] can only be allocated either none or all of the K departure links at any slot). As another example, consider the scenario that the service capacity is proportional to the available channel capacity/bandwidth which may fluctuate over time (e.g., in a wireless communications network, the channel capacity/bandwidth fluctuates over time due to many effects such as fading). In such a scenario, the service capacity of the queue is time varying in nature as it reflects the fluctuation of the channel capacity/bandwidth over time.

B. Intrinsic Properties of Buffering Tags

Consider the network element in Figure 1. We introduce *tags* and *buffering tags* that will be used in our priority-based routing policy in Section II-C. A packet p in the network element at slot t is assigned a unique positive integer $\tau_p(t)$, called the tag of packet p at slot t, to indicate its priority level so that the i^{th}-highest-priority packet in the network element at slot t has a tag equal to i. Thus, the $q(t-1)+a(t)$ packets in the network element at slot t are assigned tags from 1 to $q(t-1)+a(t)$ in the order of decreasing priority (note that a packet with smaller tag has higher priority than a packet with larger tag). Furthermore, a packet p that has to be buffered in the network element at slot t is assigned a unique positive integer $\widetilde{\tau}_p(t)$, called the buffering tag of packet p at slot t, so that the i^{th}-highest-priority packet among all of the packets that have to be buffered in the network element at slot t has a buffering tag equal to i. Therefore, the $q(t-1)+a(t)-d(t)-\ell(t)$ packets that have to be buffered in the network element at slot t are assigned buffering tags from 1 to $q(t-1)+a(t)-d(t)-\ell(t)$ in the order of decreasing priority (note that a packet with smaller buffering tag has higher priority than a packet with larger buffering tag).

Consider a packet p in the network element at slot t (note that packet p is either a packet buffered in the network element at slot $t-1$ or a packet from one of the arrival links at slot t). We denote $a_p(t)$ (resp., $d_p(t)$ and $\ell_p(t)$) as the number of arrival (resp., departure and loss) packets at slot t with priorities higher than packet p.

(i) If packet p has to be buffered in the network element at slot t and the properties (P4) and (P5) are satisfied at slot t, then we have $d_p(t)=d(t)$ and $\ell_p(t)=0$, and hence it is clear from the definition of buffering tag that $\widetilde{\tau}_p(t)=\tau_p(t)-d(t)$.

(ii) If packet p is buffered in the network element at slot $t-1$ and the property (P1) is satisfied at slot $t-1$, then there is no internal packet loss in the network element at slot $t-1$ and hence it follows that $\tau_p(t)=\widetilde{\tau}_p(t-1)+a_p(t)$.

(iii) If packet p is buffered in the network element at slot $t-1$ and has to be buffered in the network element at slot t, the property (P1) is satisfied at slot $t-1$, and the properties (P4) and (P5) are satisfied at slot t, then it follows from (i) and (ii) above that

$$\widetilde{\tau}_p(t)=\widetilde{\tau}_p(t-1)+a_p(t)-d(t). \tag{1}$$

In the following, we derive two intrinsic properties of buffering tags for the network element in Figure 1.

Theorem 2 (Change of a buffering tag in a slot) *Consider the network element in Figure 1. Suppose that a packet p is buffered in the network element at slot $t-1$ and has to be buffered in the network element at slot t, and suppose that the property (P1) is satisfied at slot $t-1$ and the properties (P4) and (P5) are satisfied at slot t. Then we have*

$$-K \leq \widetilde{\tau}_p(t)-\widetilde{\tau}_p(t-1) \leq N. \tag{2}$$

Proof. It is easy to see that (2) follows from (1), $0 \leq a_p(t) \leq N$, and $0 \leq d(t) \leq K$ ■

Theorem 3 (Change of the difference between two buffering tags in a slot) *Consider the network element in Figure 1. Suppose that two packets, say packet p_1 and packet p_2, are buffered in the network element at slot $t-1$ and have to be buffered in the network element at slot t, where packet p_1 has higher priority than packet p_2, i.e., $\widetilde{\tau}_{p_1}(t-1)<\widetilde{\tau}_{p_2}(t-1)$, and suppose that the property (P1) is satisfied at slot $t-1$ and the properties (P4) and (P5) are also satisfied at slot t. Then we have*

$$0 \leq [\widetilde{\tau}_{p_2}(t)-\widetilde{\tau}_{p_1}(t)]-[\widetilde{\tau}_{p_2}(t-1)-\widetilde{\tau}_{p_1}(t-1)] \leq N. \tag{3}$$

Proof. It is easy to see that (3) follows from (1) and $0 \leq a_{p_2}(t)-a_{p_1}(t) \leq N$. ■

Remark 4 *We note that the results in Theorem 2 and Theorem 3 are intrinsic properties of buffering tags, and they are independent of how an N-to-K priority queue is implemented. Specifically, they apply to any N-to-K priority queue that satisfies the properties (P1), (P4), and (P5). Furthermore, they could possibly be useful in devising better constructions of optical N-to-K priority queues in future works.*

C. The Priority-Based Routing Policy

The idea behind the priority-based routing policy is to route packets at the input links of the crossbar switch in Figure 2 that have to be buffered in the queue to the k groups of nFM1's according to their buffering tags.

For this, we let U_k be the targeted buffer size of the optical N-to-K priority queue in our construction, and we associate the i^{th} group of nFM1's in Figure 2 with a unique set $\Psi_i=\{U_{i-1}+1, U_{i-1}+2, \ldots, U_i\}$ of buffering tags for $i=1,2,\ldots,k$, where

$$U_0=0<U_1<U_2<\cdots<U_k. \tag{4}$$

It is clear that the sets $\Psi_1, \Psi_2, \ldots, \Psi_k$ form a partition of the set $\{1,2,\ldots,U_k\}$ of buffering tags, and we have $|\Psi_i|=U_i-U_{i-1}$ and $U_i=\sum_{j=1}^{i}|\Psi_j|$ for $i=1,2,\ldots,k$. Let $L_i=U_{i-1}+1$ so that $L_i \leq U_i$ and we can write Ψ_i as $\Psi_i=\{L_i, L_i+1, \ldots, U_i\}$ for $i=1,2,\ldots,k$.

Then the crossbar switch in Figure 2 is operated according to the following priority-based routing policy at all slots $t \geq 1$, where the parameters s_1 and s_2 satisfy $1 \leq s_1, s_2 \leq k-1$.

(R1) Departure packets: If there are packets in the queue at slot t and the service capacity of the queue at slot t is not zero, i.e., $q(t-1)+a(t)>0$ and $c(t)>0$, then the $\min\{q(t-1)+a(t), c(t)\}$ highest-priority packets (if any) among all of the packets from the N arrival links or the $m(s_1+1)$ output links of the first s_1+1 groups of nFM1's at slot t are routed to departure links $1,2,\ldots,\min\{q(t-1)+a(t), c(t)\}$ in the order of decreasing prioritie. Otherwise, no packets are routed to the departure links at slot t.

(R2) Loss packets: If there is a buffer overflow at slot t, i.e., $q(t-1)+a(t)-c(t)>B$, then the $q(t-1)+a(t)-c(t)-U_k$ lowest-priority packets (if any) among all of the packets from the N arrival links or the $m(s_2+1)$ output links of the last s_2+1 groups of nFM1's at slot t are routed to loss links $1,2,\ldots,q(t-1)+a(t)-c(t)-U_k$ in the order of decreasing priorities. Otherwise, no packets are routed to the loss links at slot t.

(R3) Round-robin routing at the k groups of nFM1's: A packet p at the input links of the crossbar switch that has to be buffered in the queue and has a buffering tag $\widetilde{\tau}_p(t) \in \Psi_i$

is routed to the i^{th} group of nFM1's. Furthermore, packets routed to a group of nFM1's are distributed to the nFM1's in that group in a *round-robin* fashion (see [8] for details) so that load balancing among the nFM1's in that group can be achieved and hence the buffering capacity of the nFM1's can be fully utilized.

In the following, we use the intrinsic properties in Section II-B to derive two basic properties of buffering tags under our priority-based routing policy. Due to space limit, their proofs can be found in the full version of this paper [16].

Theorem 5 (Range of the buffering tags at each group of nFM1's) *Assume that the feedback system in Figure 2 is operated under the routing policy (R1)–(R3) at all slots, the property (P1) is satisfied up to slot $t-1$, and the properties (P4) and (P5) are satisfied up to slot t. Suppose that a packet p is buffered in the i^{th} group of nFM1's at slot t for some $1 \leq i \leq k$. Then we have*

$$L_i - K(B_i - 1) \leq \widetilde{\tau}_p(t) \leq U_i + N(B_i - 1). \tag{5}$$

Theorem 6 (Number of packets buffered in or routed to each group of nFM1's in a slot) *Assume that the feedback system in Figure 2 is operated under the routing policy (R1)–(R3) at all slots, the property (P1) is satisfied up to slot $t-1$, and the properties (P4) and (P5) are satisfied up to slot t. Suppose that two packets, say packet p_1 and packet p_2, are buffered in or routed to the i^{th} group of nFM1's at slot t for some $1 \leq i \leq k$. Then we have*

$$|\widetilde{\tau}_{p_1}(t) - \widetilde{\tau}_{p_2}(t)| \leq |\Psi_i| + \max\{N, K\}(B_i - 1) - 1. \tag{6}$$

Therefore, there are at most $|\Psi_i| + \max\{N, K\}(B_i - 1)$ packets buffered in or routed to the i^{th} group of nFM1's at slot t.

D. The Main Results

In this section, we use the intrinsic and basic properties of buffering tags to show that the feedback system in Figure 2 can be operated as an optical N-to-K priority queue with time-varying service capacity under the routing policy (R1)–(R3). The proof of the following theorem can be found in [16].

Theorem 7 *Assume that the feedback system in Figure 2 is operated under the routing policy (R1)–(R3) at all slots. Suppose that $1 \leq s_1, s_2 \leq k-1$, $m \geq \max\{N, K\}$, and $n, B_1, B_2, \ldots, B_k, |\Psi_1|, |\Psi_2|, \ldots, |\Psi_k|$ satisfy the following conditions (A1)–(A3):*

(A1) $n \geq \min\{s_1 + s_2 + 1, k\} + 1$.

(A2) $B_i \geq 1$ *for* $i = 1, 2, \ldots, k$,

$$KB_i \leq \begin{cases} U_{i-1} + K, & \text{if } 1 \leq i \leq s_1 + 1, \\ U_{i-1} - U_{i-s_1-1}, & \text{if } s_1 + 2 \leq i \leq k, \end{cases}$$

and

$$NB_i \leq \begin{cases} U_{i+s_2} - U_i, & \text{if } 1 \leq i \leq k - s_2 - 1, \\ U_k - U_i + N, & \text{if } k - s_2 \leq i \leq k. \end{cases}$$

Note that we can see from the above inequalities that $B_1 = B_k = 1$. Also recall that $U_i = \sum_{j=1}^{i} |\Psi_j|$ for $i = 1, 2, \ldots, k$.

(A3) $1 \leq |\Psi_i| \leq (m - \max\{N, K\})B_i + \max\{N, K\}$ *for* $i = 1, 2, \ldots, k$.

Then the feedback system in Figure 2 can be operated as an optical N-to-K priority queue with time-varying service capacity $c(t)$ and buffer size U_k at all slots $t \geq 1$.

Remark 8 *We note that when $N = K = 1$ and $s_1 = s_2 = s$, the condition (A1) becomes $n \geq \min\{2s+1, k\} + 1$, the condition (A2) becomes $B_1 = B_k = 1$, $B_i \geq 1$ for $i = 2, 3, \ldots, k$,*

$$B_i \leq \begin{cases} U_{i-1} + 1, & \text{if } 2 \leq s + 1, \\ U_{i-1} - U_{i-s-1}, & \text{if } s + 2 \leq i \leq k, \end{cases}$$

and

$$B_i \leq \begin{cases} U_{i+s} - U_i, & \text{if } 1 \leq i \leq k - s - 1, \\ U_k - U_i + 1, & \text{if } k - s \leq i \leq k - 1, \end{cases}$$

and the condition (A3) becomes $1 \leq |\Psi_i| \leq (m-1)B_i + 1$ for $i = 1, 2, \ldots, k$. It is easy to see that the conditions (A1) and (A3) in [8] are the same as the conditions (A1) and (A3) above, and the condition (A2) in [8] requires that $B_i \leq U_{i-1}$ for $2 \leq i \leq s+1$ and $B_i \leq U_k - U_i$ for $k-s \leq i \leq k-1$ so that it is more restrictive than the condition (A2) above. Therefore, when $N = K = 1$ and $s_1 = s_2 = s$, our constructions in Theorem 7 subsume those in [8] as special cases.

For the special case that $N = K$, $s_1 = s_2 = s$, and m is an integer multiple of N, we show that by using an optical $(M + 2N) \times (M + 2N)$ (bufferless) crossbar switch and M fiber delay lines, we can construct an optical N-to-N priority queue with buffer size $U_k = 2^{O(\sqrt{\alpha M/N})}$, where α is a constant that depends on s, k, and m (see [16] for details). Our result (exponential in $\sqrt{M/N}$) substantially improves the best known result (polynomial in M/N) in [9].

III. Conclusion

In this paper, we obtained a class of SDL constructions of optical priority queues with multiple inputs/outputs and time-varying service capacity under our priority-based routing policy. We derived two intrinsic properties and two basic properties of buffering tags and used them to show that the feedback system in Figure 2 can be operated as an optical N-to-K priority queue with time-varying service capacity under our priority-based routing policy. We also showed that by using an optical $(M + 2N) \times (M + 2N)$ (bufferless) crossbar switch and M fiber delay lines, an optical N-to-N priority queue with buffer size $2^{O(\sqrt{M/N})}$ (exponential in $\sqrt{M/N}$) can be constructed, which greatly improves the best known result $O(M^3/N^2)$ (polynomial in M/N) in the literature.

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