Covariance-Based Time-Frequency ESPRIT Algorithm for Direction-of-Arrival Estimation

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***Abstract*—In this paper, a new version of time-frequency (T-F) ESPRIT algorithm with reduced computational complexity is proposed. The key idea of proposed covariance-based T-F ESPRIT (CB T-F ESPRIT) algorithm is to use the covariance-based DoA (CB-DoA) approach for the signal subspace construction. Specifically, the proposed CB T-F ESPRIT algorithm first constructs the time-frequency data model and then exploits the STFD matrix for the estimation of signal subspace. In particular, instead of directly performing EVD on the covariance matrix obtained from the averaged STFD matrix, the proposed scheme employs the CB-DoA approach which provides a lower computational complexity while maintaining the performance gain of T-F ESPRIT algorithm over the conventional ESPRIT algorithm. From the computational complexity analysis and the numerical evaluations, we demonstrate that CB T-F ESPRIT algorithm outperforms the conventional DoA estimation schemes with reduced computational complexity.**

I. Introduction

Direction of arrival (DoA) estimation has received much attention over the past few years [1]–[4]. Although the global positioning system (GPS) provides fast and real-time localization service, it might not available in the indoor environment and cannot provide high accuracy. As a remedy, various DoA estimation techniques have been proposed in recent years [5]–[7]. One of the most popular and widely used DoA estimation technique is multiple signal classification (MUSIC) algorithm [5]. MUSIC algorithm is a type of subspace method that decomposes the eigenspace of signal covariance matrix into the signal subspace and the noise subspace and then exploits the orthogonality between two subspaces for the DoA estimation. While the MUSIC algorithm can accurately estimate the direction of multiple signals, it has a fundamental limitation that the resolution of angle estimation is limited. To overcome this issue, an estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm has been proposed [6]. Also in [7], time-frequency (T-F) ESPRIT algorithm using the spatial time-frequency distribution (STFD) matrix has been proposed. The T-F ESPRIT algorithm can improve the DoA estimation accuracy over the conventional ESPRIT algorithm but the computational complexity is burdensome due to the eigenvalue decomposition (EVD).

In this paper, a new version of T-F ESPRIT algorithm with reduced computational complexity is proposed. The key idea of proposed covariance-based T-F ESPRIT (CB T-F ESPRIT) algorithm is to use the covariance-based DoA (CB-DoA) approach for the signal subspace construction [8], [9]. Specifically, the proposed CB T-F ESPRIT algorithm first constructs the time-frequency data model and then exploits the STFD matrix for the estimation of signal subspace. In particular, instead of directly performing EVD on the covariance matrix obtained from the averaged STFD matrix, the proposed scheme employs the CB-DoA approach which provides a lower computational complexity while maintaining the performance gain of T-F ESPRIT algorithm over the conventional ESPRIT algorithm. From the computational complexity analysis and the numerical evaluations, we demonstrate that CB T-F ESPRIT algorithm outperforms the conventional DoA estimation schemes with reduced computational complexity.

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II. System Model

We consider a multiple-input multiple-output (MIMO) systems with P transmitting sources and $2M$ uniform linear array (ULA) of receiving antennas ($M > P$) [10], [11]. The receiving antennas are grouped in doublets with displacement Δ and the antennas in each doublet have a constant displacement d (see Fig. 1). We focus on chirp and analytic narrowband frequency modulation (FM) signal, which is modulated as

$$\mathbf{s}(t) = [s_1(t) \cdots s_P(t)]^{\mathrm{T}} \tag{1}$$
$$= \left[s_1 e^{j\psi_1(t)} \cdots s_P e^{j\psi_P(t)}\right]^{\mathrm{T}}, \tag{2}$$

where s_p and $\psi_p(t)$ are the fixed amplitude and the time-varying phase of the p-th source signal with the impinging angle θ_p, respectively. We also assume that the transmitted signals propagate in a stationary environment and are mutually uncorrelated over the T observations. Under this assumption, we get

$$\mathbb{E}\left[s_i(t)s_j^*(t)\right] = \delta_{i,j}. \tag{3}$$

Let $x_m(t)$ and $y_m(t)$ be the received signals of the m-th antenna of the first and second antenna doublet at time t as

$$x_m(t) = \sum_{p=1}^{P} s_p(t) a_m(\theta_p) + n_{x,m}(t), \tag{4}$$

$$y_m(t) = \sum_{p=1}^{P} s_p(t) e^{-j\frac{w\Delta \sin\theta_p}{c}} a_m(\theta_p) + n_{y,m}(t), \tag{5}$$

where $a_m(\theta_p) = e^{-j(m-1)\frac{wd\sin\theta_p}{c}}$ is the phase delay, w is the central frequency, c is the propagation speed, and $n_{x,m}(t)$ and $n_{y,m}(t)$ are the noise components.

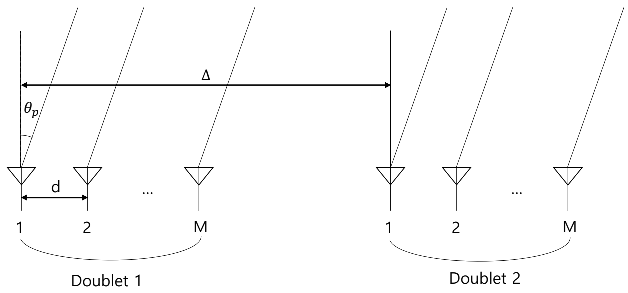

Fig. 1: ULA Array structure with 10 elements in each doublet.

Now let $\mathbf{x}(t)$ and $\mathbf{y}(t)$ be the received signal vectors of each doublet at time t:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}_x(t), \tag{6}$$
$$\mathbf{y}(t) = \mathbf{A}\boldsymbol{\Phi}\mathbf{s}(t) + \mathbf{n}_y(t), \tag{7}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1)\cdots\mathbf{a}(\theta_P)]$ is the array steering matrix which has $\mathbf{a}(\theta_p) = [a_1(\theta_p)\cdots a_M(\theta_p)]^{\mathrm{T}}$ as its p-th column vector, $\mathbf{n}_x(t) = [n_{x,1}(t)\cdots n_{x,M}(t)]$, and $\mathbf{n}_y(t) = [n_{y,1}\cdots n_{y,M}]$. The matrix $\boldsymbol{\Phi}$ is a $P\times P$ diagonal matrix whose diagonal elements are the phase delay between the two doublets given by

$$\boldsymbol{\Phi} = \mathrm{diag}\{e^{-j\frac{w\Delta\sin\theta_1}{c}},\cdots,e^{-j\frac{w\Delta\sin\theta_P}{c}}\}. \tag{8}$$

By grouping the doublet received signals into $\mathbf{z}(t) = \begin{bmatrix}\mathbf{x}(t)\\ \mathbf{y}(t)\end{bmatrix}$, the transmission modeling becomes

$$\mathbf{z}(t) = \bar{\mathbf{A}}\mathbf{s}(t) + \bar{\mathbf{n}}(t), \tag{9}$$

where $\bar{\mathbf{A}} = \begin{bmatrix}\mathbf{A}\\ \mathbf{A}\boldsymbol{\Phi}\end{bmatrix}$ and $\bar{\mathbf{n}}(t) = \begin{bmatrix}\mathbf{n}_x(t)\\ \mathbf{n}_y(t)\end{bmatrix}$. We assume that the noise is spatially and temporally white zero mean process. That is

$$\mathbb{E}\big[\bar{\mathbf{n}}(t)\bar{\mathbf{n}}^{\mathrm{H}}(s)\big] = \sigma^2\mathbf{I}_{2M}\delta_{t,s}, \tag{10}$$

where σ^2 is the noise variance.

III. Spatial Time-Frequency Distribution

Conventional approaches use the signal covariance matrix to compute the signal subspace only in the time domain. In contrast, the time-frequency method deals with the time-frequency distribution (TFD) of the specific temporal windows for the specific frequencies which can fully exploit the time-frequency characteristics of signals. The discrete Spatial Pseudo-Wigner-Ville distribution (SPWVD) matrix is given by

$$\hat{\mathbf{D}}_{zz}(t,f) = \sum_{\tau=-(L-1)/2}^{(L-1)/2}\mathbf{z}(t+\tau)\mathbf{z}^{\mathrm{H}}(t-\tau)e^{-j4\pi f\tau}, \tag{11}$$

where L is the odd window length. The data matrix formed by the received signal vector is specifically shown as

$$\mathbf{Z}(t) = \left[\mathbf{z}\left(t-\frac{L-1}{2}\right)\cdots\mathbf{z}\left(t+\frac{L-1}{2}\right)\right] \tag{12}$$
$$= \begin{bmatrix} z_1(t-\frac{L-1}{2}) & \cdots & z_1(t+\frac{L-1}{2})\\ \vdots & \ddots & \vdots \\ z_{2M}(t-\frac{L-1}{2}) & \cdots & z_{2M}(t+\frac{L-1}{2})\end{bmatrix}. \tag{13}$$

Note that the SPWVD matrix can be re-expressed as

$$\hat{\mathbf{D}}_{zz}(t,f) = \mathbf{Z}(t)\mathbf{F}(f)\boldsymbol{\Pi}\mathbf{Z}^{\mathrm{H}}(t), \tag{14}$$

where $\boldsymbol{\Pi}$ is an $L\times L$ exchange matrix and

$$\mathbf{F}(f) = \mathrm{diag}\Big\{e^{j4\pi f\frac{L-1}{2}}, e^{j4\pi f\frac{L-3}{2}},\cdots,e^{-j4\pi f\frac{L-1}{2}}\Big\}. \tag{15}$$

Taking the expectation and assuming that the source signal and noise are statistically independent, we get

$$\mathbf{D}_{zz}(t,f) = \mathbb{E}\big[\hat{\mathbf{D}}_{zz}(t,f)\big] \tag{16}$$
$$= \bar{\mathbf{A}}\mathbf{D}_{ss}(t,f)\bar{\mathbf{A}}^{\mathrm{H}} + \sigma^2\mathbf{I}_{2M}, \tag{17}$$

where

$$\mathbf{D}_{ss}(t,f) = \mathbb{E}\big[\hat{\mathbf{D}}_{ss}(t,f)\big], \tag{18}$$
$$\hat{\mathbf{D}}_{ss}(t,f) = \sum_{\tau=-(L-1)/2}^{(L-1)/2}\mathbf{s}(t+\tau)\mathbf{s}^{\mathrm{H}}(t-\tau)e^{-j4\pi f\tau}. \tag{19}$$

The equation (18) clearly states that the STFD matrix works as a covariance matrix in the ESPRIT algorithm. The difference between STFD matrix and the covariance matrix is that the covariance matrix is time-dependent where as the STFD matrix is time and frequency-dependent.

IV. Covariance-Based T-F ESPRIT Algorithm

A. Utilization of Multiple t-f Points

In practice, the averaged STFD matrix obtained by averaging the sample SPWVD matrices over multiple time-frequency points are used instead of the true STFD matrix. Through the EVD of the averaged STFD matrix, we can obtain the signal subspace spanned by the columns of $\bar{\mathbf{A}}$ and then exploit the rotational invariance property of $\bar{\mathbf{A}}$ to estimate the signal DoAs. Let $\bar{\mathbf{D}}_{zz}$ be the STFD matrix averaged over PK time-frequency points $\{t_k, f_{p,k}(t_k)\}_{p,k}$:

$$\bar{\mathbf{D}}_{zz} = \frac{1}{PK}\sum_{p=1}^{P}\sum_{k=1}^{K}\hat{\mathbf{D}}_{zz}(t_k, f_{p,k}(t_k)) \tag{20}$$
$$= \frac{1}{PK}\sum_{p=1}^{P}\sum_{k=1}^{K}\mathbf{Z}_k\mathbf{F}_{p,k}\mathbf{J}\mathbf{Z}_k^{\mathrm{H}}, \tag{21}$$

where the intermediate frequency law of the p-th signal at the k-th time sample is $f_{p,k}(t_k)$, and $\mathbf{Z}_k = \mathbf{X}(t_k)$, and $\mathbf{F}_{p,k} = \mathbf{F}(f_{p,k}(t_k)))$.

B. Covariance-Based T-F ESPRIT Algorithm

Let $\mathbf{J}_1$ and $\mathbf{J}_2$ be selection matrices such that

$$\mathbf{J}_1 = [\mathbf{I}_M\,\mathbf{0}_M], \tag{22}$$
$$\mathbf{J}_2 = [\mathbf{0}_M\,\mathbf{I}_M]. \tag{23}$$

Using these selection matrices, the array steering matrix can be divided as

$$\begin{bmatrix}\mathbf{J}_1\bar{\mathbf{A}}\\ \mathbf{J}_2\bar{\mathbf{A}}\end{bmatrix} = \begin{bmatrix}\mathbf{A}\\ \mathbf{A}\boldsymbol{\Phi}\end{bmatrix} \tag{24}$$

From the equation (24), the rotational invariance property is obtained, i.e.

$$\mathbf{J}_2\bar{\mathbf{A}} = \mathbf{J}_1\bar{\mathbf{A}}\boldsymbol{\Phi} \tag{25}$$

The rotational invariance property is the key idea for both ESPRIT and CB-DoA algorithms in revealing Φ. Since the array steering matrix $\bar{\mathbf{A}}$ is unknown, ESPRIT algorithm uses the signal subspace matrix obtained from the eigenvalue decomposition of the covariance matrix. Since the signal subspace matrix has the same column space with $\bar{\mathbf{A}}$, it inherits the rotational invariance property. On the other hand, the CB-DoA algorithm eliminates the effect of noise and transform $\mathbf{A}$ into an unitary matrix which is much easier to handle. Hence, without knowing $\mathbf{A}$, we can still use the rotational invariance property in CB-DoA algorithm.

Considering the fact that the averaged STFD matrix in T-F ESPRIT algorithm acts as the covariance matrix in ESPRIT algorithm, it is natural to define $\mathbf{D}_1$ and $\mathbf{D}_2$ as

$$\mathbf{D}_1 = \mathbf{J}_1(\bar{\mathbf{D}}_{zz} - \sigma^2\mathbf{I}_{2M})\mathbf{J}_1^{\mathrm{H}}, \quad (26)$$

$$\mathbf{D}_2 = \mathbf{J}_2(\bar{\mathbf{D}}_{zz} - \sigma^2\mathbf{I}_{2M})\mathbf{J}_1^{\mathrm{H}}, \quad (27)$$

where σ^2 is the estimate noise power. By performing the EVD of $\mathbf{D}_1$ as

$$\mathbf{D}_1 = \mathbf{U}\mathbf{\Sigma}^2\mathbf{U}^{\mathrm{H}}, \quad (28)$$

we may form the matrices $\mathbf{\Sigma}_s^2$ and $\mathbf{U}_s$ corresponding to the P largest eigenvalues of $\mathbf{D}_1$ and their corresponding eigenvectors, respectively, such that $\mathbf{J}_1\bar{\mathbf{A}}$ satisfies

$$\mathbf{J}_1\bar{\mathbf{A}} = \mathbf{U}_s\mathbf{\Sigma}_s\mathbf{V}, \quad (29)$$

where $\mathbf{V}$ is an unitary matrix. Thus, by defining transformation $\mathbf{F}$ as

$$\mathbf{F} = \mathbf{\Sigma}_s^{-1}\mathbf{U}_s, \quad (30)$$

$\mathbf{A}$ can be transformed into an unitary matrix $\mathbf{V}$ as

$$\mathbf{F}\mathbf{J}_1\bar{\mathbf{A}} = \mathbf{V}. \quad (31)$$

Now, let $\mathbf{D}_F$ be the matrix obtained by multiplying $\mathbf{F}$ at both sides of $\mathbf{D}_2$. Applying the equation (28) and using the rotational invariance property of $\bar{\mathbf{A}}$, we obtain

$$\mathbf{D}_F = \mathbf{F}\mathbf{D}_2\mathbf{F}^{\mathrm{H}} \quad (32)$$

$$= \mathbf{F}\mathbf{J}_2\bar{\mathbf{A}}\mathbf{D}_{ss}\bar{\mathbf{A}}^{\mathrm{H}}\mathbf{J}_1^{\mathrm{H}} \quad (33)$$

$$= \mathbf{F}\mathbf{J}_1\bar{\mathbf{A}}\mathbf{\Phi}\mathbf{D}_{ss}\bar{\mathbf{A}}^{\mathrm{H}}\mathbf{J}_1^{\mathrm{H}} \quad (34)$$

$$= \mathbf{V}\mathbf{\Phi}\mathbf{V}^{\mathrm{H}}. \quad (35)$$

Hence, from the discussion above, $\mathbf{\Phi}$ can be found by performing the EVD on $\mathbf{D}_F$. Since the elements of $\mathbf{\Phi}$ have unit norm, Φ is generated with the normalized eigenvalues of $\mathbf{D}_F$ and the DoAs are calculated from the diagonal elements of $\mathbf{\Phi}$.

V. Computational Complexity Analysis

The main goal of this paper is the computational complexity reduction of the T-F ESPRIT algorithm through the covariance based approach. For that purpose, we summarize the basic operations of the proposed CB T-F ESPRIT algorithm and the conventional T-F ESPRIT algorithm in Table I. In the conventional T-F ESPRIT algorithm, Ψ is calculated as the total least square (TLS) solution. For convenience, the computational complexity of simple matrix addition and diagonal matrix operations are ignored.

From Table 1, we can verifies that the T-F ESPRIT algorithm requires:

T-F ESPRIT	CB T-F ESPRIT
$[\mathbf{E}_s, \sigma^2] = \mathrm{EVD}(\mathbf{D}_{zz})$	$[\mathbf{E}_s, \sigma^2] = \mathrm{EVD}(\mathbf{J}_1\mathbf{D}_{zz}\mathbf{J}_1^{\mathrm{H}})$
$\mathbf{E}_x = \mathbf{J}_1\mathbf{E}_s$	$\mathbf{F} = \mathbf{\Sigma}_s^{-1}\mathbf{E}_s^{\mathrm{H}}$
$\mathbf{E}_y = \mathbf{J}_2\mathbf{E}_s$	$\mathbf{D}_2 = \mathbf{J}_2(\hat{\mathbf{D}}_{zz} - \sigma^2)\mathbf{J}_1^{\mathrm{H}}$
$\mathbf{E}_a = [\mathbf{E}_x\ \mathbf{E}_y]^{\mathrm{H}}[\mathbf{E}_x\ \mathbf{E}_y]$	$\mathbf{D}_F = \mathbf{F}\mathbf{D}_2\mathbf{F}^{\mathrm{H}}$
$\mathbf{E}, \mathbf{\Lambda} = \mathrm{EVD}(\mathbf{E}_a)$	$\mathbf{\Phi} = \mathrm{EVD}(\mathbf{D}_F)$
$\begin{bmatrix}\mathbf{E}_{11}\\ \mathbf{E}_{22}\end{bmatrix} = \begin{bmatrix}\mathbf{J}_1\mathbf{E}\\ \mathbf{J}_2\mathbf{E}\end{bmatrix}$	
$\mathbf{\Psi} = -\mathbf{E}_{12}\mathbf{E}_{22}^{-1}$	
$\mathbf{\Phi} = \mathrm{EVD}(\mathbf{\Psi})$	

TABLE I: Comparison of the proposed CB T-F ESPRIT algorithm and the conventional T-F ESPRIT algorithm.

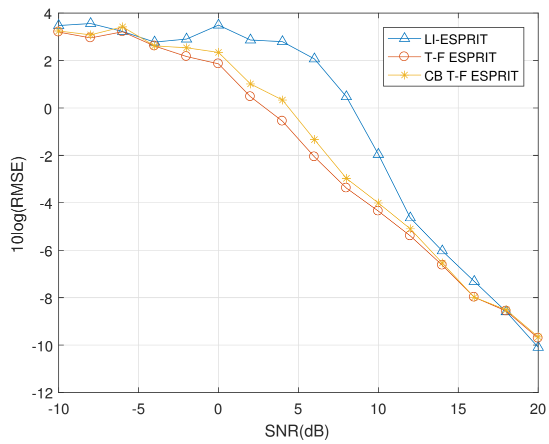

Fig. 2: RMSE versus SNR ($M = 10$, $T = 200$, $L = 32$, 500 trails per simulated point.

- PK matrix multiplications of a $2M \times L$ by a $L \times 2M$ matrix
- 1 eigenvalue decomposition of a $2M \times 2M$ matrix
- 1 matrix multiplication of a $M \times 2P$ by a $M \times 2P$ matrix
- 1 eigenvalue decomposition of a $2P \times 2P$ matrix
- 1 full-matrix inversion of a $P \times P$ matrix
- 1 matrix multiplication of a $P \times P$ by $P \times P$ matrix
- 1 eigenvalue decomposition of a $P \times P$ matrix

On the other hand, the proposed CB T-F ESPRIT algorithm requires:

- PK matrix multiplications of a $2M \times L$ by a $L \times 2M$ matrix
- 1 eigenvalue decomposition of a $M \times M$ matrix
- 3 matrix multiplication of a $P \times P$ by a $P \times M$ matrix
- 1 eigenvalue decomposition of a $P \times P$ matrix

Clearly, the computational cost of CB T-F ESPRIT algorithm is much smaller than that of T-F ESPRIT algorithm. In fact, CB T-F ESPRIT algorithm requires a half size eigenvalue decomposition compare to T-F ESPRIT algorithm which occupy most of the computational complexity.

VI. Simulation Results

We consider a ULA system with two doublet array including 10 array sensors with the spacing between each array element is $d = \frac{\lambda}{2}$ and the spacing between two doublet array is $\Delta = 10d$ (see Fig. 1). Two narrowband chirp signals impinge on the array from $\theta_1 = 24^\circ$ and $\theta_2 = 25^\circ$. The source waveforms are modeled as

$$s_1(t) = e^{2\pi(f_1 t + \beta_1 t^2/2)} \tag{36}$$

$$s_2(t) = e^{2\pi(f_2 t + \beta_2 t^2/2)} \tag{37}$$

where $f_1 = 0$ and $f_2 = \frac{1}{2}$ are the discrete-time frequencies of the two source signals while their chirp rates are chosen as $\beta_1 = 0.002$ and $\beta_2 = -0.002$. The noise model is complex Gaussian zero-mean spatially and temporally white process. The averaged STFD matrix $\bar{\mathbf{D}}$ is computed for each source signal separately by averaging the sample STFD matrices computed at 200 different time-frequency points that belong to the source signatures through time-frequency processing window with length $L = 32$. A total of 300 independent Monte-Carlo simulation runs have been used to obtain each simulated point. In order to compare the performance of the proposed CB T-F ESPRIT, we use the T-F ESPRIT and ESPRIT algorithms as benchmark schemes [6], [7]. As a performance metric, we use root means square errors (RMSE) defined as

$$\text{RMSE} = \frac{1}{I}\sum_{i=1}^{I}\sqrt{\sum_{p=1}^{P}(\hat{\theta}_{p,i} - \theta_{p,i})^2} \tag{38}$$

where I is the number of Monte-Carlo simulation, $\hat{\theta}_{p,i}$ and $\theta_{p,i}$ are the p-th estimated DoA and the real DoA at i-th Monte-Carlo simulation, respectively. A logarithmic form of RMSE is $10\log \text{RMSE}$.

In Fig. 2, we plot the RMSE as a function of signal-to-noise-ratio (SNR). We observe that the proposed CB T-F ESPRIT algorithm outperforms the conventional ESPRIT algorithm. Even when compared with the T-F ESPRIT algorithm, the proposed scheme achieves a similar performance. In Fig. 3, we fix the SNR to 10 dB and plot the RMSE as a function of the number of snapshots. One can observe from Fig. 3 that the CB T-F ESPRIT algorithm has almost the same performance with the T-F ESPRIT algorithm and much better performance than the conventional ESPRIT algorithm.

VII. Conclusion

In this paper, we proposed a novel DoA estimation algorithm with reduced computational complexity. The key idea of proposed CB T-F ESPRIT algorithm is to use the CB-DoA approach for the signal subspace construction. To be specific, instead of directly performing EVD on the covariance matrix obtained from the averaged STFD matrix, the proposed scheme employs the CB-DoA approach which provides a lower computational complexity while maintaining the performance gain of T-F ESPRIT algorithm over the conventional ESPRIT algorithm. From the computational complexity analysis and the numerical evaluations, we demonstrate that CB T-F ESPRIT algorithm outperforms the conventional DoA estimation schemes with reduced computational complexity.

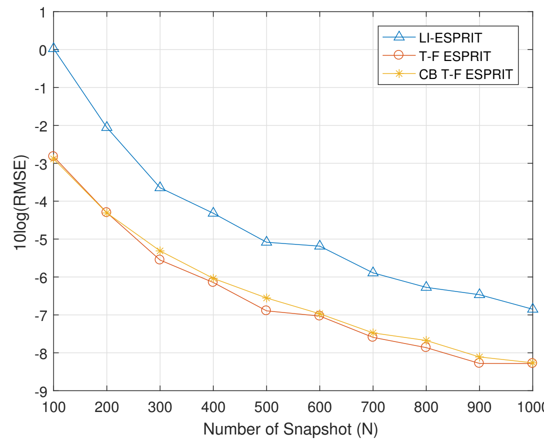

Fig. 3: RMSE versus the number of snapshots ($M = 10$, SNR $= 10$ dB, $L = 32$, 500 trails per simulated point.

References

[1] S. Kim, S. Park, H. Ji, and B. Shim, "AOA-TOA based localization for 5G cell-less communications," in *Proc. IEEE 23rd Asia-Pacific Conf. on Commun. (APCC)*, Dec. 2017.

[2] D. Chen, S. Shi, X. Gu, and B. Shim, "Robust DoA Estimation Using Denoising Autoencoder and Deep Neural Networks," *IEEE Access*, 2022.

[3] S. Kim and B. Shim, "FDD-based cell-free massive MIMO systems," in *Proc. IEEE 19th Int. Workshop on Sig. Process. Advances in Wireless Commun. (SPAWC)*, June. 2018.

[4] R. Zhang, B. Shim, and W. Wu, "Direction-of-Arrival Estimation for Large Antenna Arrays With Hybrid Analog and Digital Architectures," *IEEE Trans. on Sig. Process.*, vol. 70, pp. 72–88, 2021.

[5] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas. Propagat.*, vol. 34, no. 3, pp. 276–280, 1986.

[6] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust. Speech. Signal. Process.*, vol. 37, no. 7, pp. 984–995, 1989.

[7] A. Hassanien, A. B. Gershman, and M. G. Amin, "Time-frequency ESPRIT for direction-of-arrival estimation of chirp signals," in *Sensor Array and Multichannel Signal Processing Workshop Proceedings, 2002*. IEEE, Aug.

[8] T. N. Ferreira, S. L. Netto, and P. S. Diniz, "Low complexity covariance-based DoA estimation algorithm," in *Proc. IEEE 15th European Sig. Process. Conf.*, Sep. 2007.

[9] ——, "Direction-of-arrival estimation using a low-complexity covariance-based approach," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 3, pp. 1924–1934, 2012.

[10] S. Kim, J. W. Choi, and B. Shim, "Downlink pilot precoding and compressed channel feedback for FDD-based cell-free systems," *IEEE Trans. on Wireless Commun.*, vol. 19, no. 6, pp. 3658–3672, 2020.

[11] S. Kim and B. Shim, "Energy-efficient millimeter-wave cell-free systems under limited feedback," *IEEE Trans. on Commun.*, vol. 69, no. 6, pp. 4067–4082, 2021.