Outage Performance Analysis of Hybrid Relay-Reconfigurable Intelligent Surface Networks

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***Abstract*—This paper investigates the wireless communications assisted by both decode-forward (DF)-relay and reconfigurable intelligent surface (RIS). For assessment of outage performance, the statistical characteristics of signal-to-noise ratios (SNRs) are investigated. For RIS-aided systems with practical design constraints, e.g., total power and the number of reflecting elements, the deployment of a relay can improve the performance. The numerical results are presented to demonstrate the performance comparison of the hybrid relay-RIS (R-RIS) and RIS-only systems under various design parameters.**

***Index Terms*—Relay-RIS systems, SNR, Outage performance**

I. INTRODUCTION

Reconfigurable intelligent surface (RIS) is a physical-layer technology to achieve high spectral and energy efficiency. Specifically, a large number of passive reflecting elements reflects the signal with controllable phase shifts without complex encoding/decoding and radio frequency processing techniques. Through intelligent placement and beamforming design, a RIS can overcome the unfavorable propagation conditions (e.g., blockage and fading) and can bring several benefits to the wireless paradigm [1], [2].

The relay and multi-antenna systems improve the signal-to-noise ratio (SNR) for the users. In this paper, we consider the downlink communications amalgamating the benefits of both a relay and RIS, and derive an approximate expression of the outage performance of hybrid relay-RIS (R-RIS) system. In particular, the statistical characteristics of SNRs are presented using a gamma distribution for effective channel powers via moment-matching technique. Finally, the numerical results are presented to validate the analytical expressions, show the impact of different parameters, and provide the performance comparison of the R-RIS and RIS-only systems.

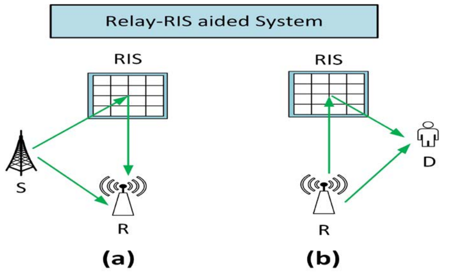

Fig. 1. System model: (a) first time slot (b) second time slot.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a time-division half-duplex downlink transmission scenario for a network consisting of a source (S), decode-and-forward (DF) relay (R), RIS, and a destination (D). The direct link between S and D does not exist. In the first time slot, S transmits its signal to both RIS and R. In the second time slot, R transmits the decoded signal to RIS and D. A RIS with M reflecting elements can adjust the phase shifts optimally to add the signals at R in the first time slot and at the D in the second time slot. The tractable independent and identically distributed (i.i.d.) Rayleigh fading model provides an upper bound performance of the general correlated Rayleigh fading [3], [4]. Specifically, $\mathbf{h}_{S.RIS} \in \mathbb{C}^M$, $\mathbf{h}_{RIS.R} \in \mathbb{C}^M$, $h_{S.R} \in \mathbb{C}$, $\mathbf{h}_{R.RIS} \in \mathbb{C}^M$, $\mathbf{h}_{RIS.D} \in \mathbb{C}^M$, and $h_{R.D} \in \mathbb{C}$ represent the channels between $S \rightarrow RIS$, $RIS \rightarrow R$, $S \rightarrow R$, $R \rightarrow RIS$, $RIS \rightarrow D$, and $R \rightarrow D$, respectively. The Rayleigh distributed channel amplitudes have a parameter $\frac{\lambda_i}{2}$, $i \in \{S.RIS, RIS.R, S.R, R.RIS, RIS.D, R.D\}$. Given P_S and P_R as the transmit powers at S and R, respectively, σ^2 as the variance of the AWGN both at R and D, and ideal RIS[1] with perfect channel state information (CSI)[2], the received

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[1] An RIS with the continuous phase and amplitude variations and the phase-independent amplitude variations provides theoretical upper bound performance for the practical RIS systems.

[2] The channel estimation of RIS systems with an affordable overhead is a non-trivial task. The quasi-static and low-dimensional properties of the channels can be exploited [5].

SNRs at the R and D in the first and second time-slots are respectively expressed as,

$$\gamma_1=\rho_S\left(|h_{S.R}|+\sum_{m=1}^{M}|\left[\mathbf{h}_{S.RIS}\right]_m\left[\mathbf{h}_{RIS.R}\right]_m|\right)^2 \tag{1}$$

$$\gamma_2=\rho_R\left(|h_{R.D}|+\sum_{m=1}^{M}|\left[\mathbf{h}_{R.RIS}\right]_m\left[\mathbf{h}_{RIS.D}\right]_m|\right)^2 \tag{2}$$

where $\rho_S=\frac{P_S}{\sigma^2}$ and $\rho_R=\frac{P_R}{\sigma^2}$.

Then, the achievable rate for the hybrid R-RIS systems can be defined as,

$$R=\frac{1}{2}\log_2\left(1+\min\left\{\gamma_1,\gamma_2\right\}\right) \tag{3}$$

By Setting $\rho=\rho_S+\rho_R$ for the performance comparison, the achievable rate for the RIS-only systems can be written as $R_0=\log_2\left(1+\rho\left(\sum_{m=1}^{M}|\left[\mathbf{h}_{S.RIS}\right]_m\left[\mathbf{h}_{RIS.D}\right]_m|\right)^2\right)$.

For hybrid R-RIS systems, the outage probability can be computed as,

$$P_{OUT}=Pr\left(\min\left\{\gamma_1,\gamma_2\right\}\le\epsilon\right)=F_{\min\{\gamma_1,\gamma_2\}}\left(\epsilon\right) \tag{4}$$

where $\epsilon=2^{2\bar{R}}-1$ and $\bar{R}$ is the data rate of the transmission.

The outage probability can be simplified as,

$$P_{OUT}=1-\left\{\left(1-F_{\gamma_1}\left(\epsilon\right)\right)\left(1-F_{\gamma_2}\left(\epsilon\right)\right)\right\} \tag{5}$$

To derive a closed-form analytical expression of the outage probability, we need to first determine the distributions of $\Omega_1=|h_{S.R}|+\sum_{m=1}^{M}|\left[\mathbf{h}_{S.RIS}\right]_m\left[\mathbf{h}_{RIS.R}\right]_m|$ and $\Omega_2=|h_{R.D}|+\sum_{m=1}^{M}|\left[\mathbf{h}_{R.RIS}\right]_m\left[\mathbf{h}_{RIS.D}\right]_m|$. Ω_1 and Ω_2 can be approximated with the Gamma RVs having the shape parameters $k_1=\frac{(\mathbb{E}\{\Omega_1\})^2}{\mathbb{V}\{\Omega_1\}}$ and $k_2=\frac{(\mathbb{E}\{\Omega_2\})^2}{\mathbb{V}\{\Omega_2\}}$ and the scale parameters $\theta_1=\frac{\mathbb{V}\{\Omega_1\}}{\mathbb{E}\{\Omega_1\}}$ and $\theta_2=\frac{\mathbb{V}\{\Omega_2\}}{\mathbb{E}\{\Omega_2\}}$, respectively, where

$$\begin{aligned}\mathbb{E}\left\{\Omega_1\right\}=&\frac{\sqrt{\lambda_{S.R}\pi}}{2}+\\&M\frac{\pi^2}{\left(16-\pi^2\right)}\left(\frac{\sqrt{\lambda_{S.RIS}}\sqrt{\lambda_{RIS.R}}\left(16-\pi^2\right)}{4\pi}\right)\end{aligned} \tag{6}$$

$$\begin{aligned}&\mathbb{V}\left\{\Omega_1\right\}=\lambda_{S.R}+\\&M\frac{\pi^2}{16-\pi^2}\left(\frac{\sqrt{\lambda_{S.RIS}}\sqrt{\lambda_{RIS.R}}\left(16-\pi^2\right)}{4\pi}\right)^2-\frac{\lambda_{S.R}\pi}{4}\end{aligned} \tag{7}$$

$$\begin{aligned}\mathbb{E}\left\{\Omega_2\right\}=&\frac{\sqrt{\lambda_{R.D}\pi}}{2}+\\&M\frac{\pi^2}{\left(16-\pi^2\right)}\left(\frac{\sqrt{\lambda_{R.RIS}}\sqrt{\lambda_{RIS.D}}\left(16-\pi^2\right)}{4\pi}\right)\end{aligned} \tag{8}$$

$$\begin{aligned}&\mathbb{V}\left\{\Omega_2\right\}=\lambda_{R.D}+\\&M\frac{\pi^2}{16-\pi^2}\left(\frac{\sqrt{\lambda_{R.RIS}}\sqrt{\lambda_{RIS.D}}\left(16-\pi^2\right)}{4\pi}\right)^2-\frac{\lambda_{R.D}\pi}{4}\end{aligned} \tag{9}$$

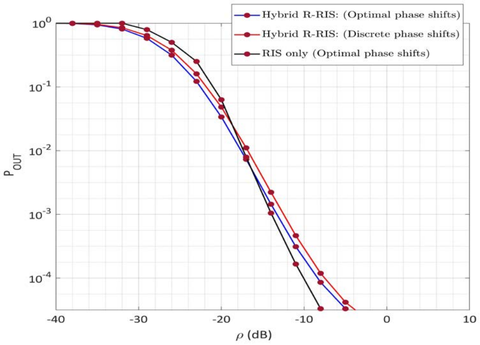

Fig. 2. Outage Probability vs. Transmit SNR (dB).

Using Eqs. (6)-(9), we can find out $F_{\gamma_1}\left(\epsilon\right)$ and $F_{\gamma_2}\left(\epsilon\right)$ as,

$$F_{\gamma_1}\left(\epsilon\right)=1-\frac{1}{\Gamma\left(k_1\right)}\Gamma\left(k_1,\frac{\sqrt{\epsilon}}{\sqrt{\rho_S}\theta_1}\right) \tag{10}$$

$$F_{\gamma_2}\left(\epsilon\right)=1-\frac{1}{\Gamma\left(k_2\right)}\Gamma\left(k_2,\frac{\sqrt{\epsilon}}{\sqrt{\rho_R}\theta_2}\right) \tag{11}$$

respectively, where $\Gamma\left(.\right)$ and $\Gamma\left(.,.\right)$ are Gamma and incomplete Gamma functions, respectively.

III. NUMERICAL RESULTS

For the numerical results shown in Fig. 2, the parameters are set as; σ^2 = 0.95, λ_i = 0.75, M = 30, $\bar{R}$ = 2 b/s/Hz. The results validate the theoretical analysis. At low- SNR, the R-RIS system outperforms the RIS-only system and validates the importance of deployment of a DF-Relay in RIS systems. However, RIS-only system performs better than R-RIS system when the SNR is very high. The same performance can be achieved for the number of reflecting elements also, i.e., R-RIS and RIS-only systems outperform better respectively when the number of reflecting elements is limited and extremely large.

IV. CONCLUSION

We investigate the wireless downlink communications for the R-RIS and RIS-only systems. For a performance metric, we derive the approximate expression of the outage performance. Using the gamma distribution, the statistics of the effective channels for S-RIS-R and S-R in first and R-RIS-D and R-D in second time slots are determined via moment-matching method. The results suggest that a optimal selection of the hybrid R-RIS and R-only systems is possible under the practical scenarios of the total power and number of reflecting elements.

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