Probability of Bit Error of MRC Detection in OFDM-MMIMO Systems Utilizing Gaussian Mixture Model

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***Abstract*—This paper derives probability distribution functions (PDFs) of the co-channel interference (CCI) and the effective noise (EN) of maximal-ratio combining (MRC) detection in orthogonal frequency-division multiplexing (OFDM) based massive multiple-input, multiple-output (MMIMO) systems. The effects of CCI and EN are expressed as a function of random variables and their PDFs for the OFDM-MMIMO system utilizing Gray-coded, binary and quadrature phase shift keying (BPSK, QPSK), and *M*-ary quadrature amplitude modulation (QAM) are derived. In addition, we firstly propose asymptotic closed-form expressions for the PDFs of the CCI and the EN in terms of Gaussian mixture model (GMM), i.e., a sum of multiple Gaussian distributions. Furthermore, the proposed PDFs are utilized to determine the probability of bit error (PBE) of the OFDM-MMIMO system. The derived equations are evaluated through Monte-Carlo simulations, and the results confirm that the derived equations produced accurate PDFs and PBE performances of OFDM-MMIMO systems. The PBE from the proposed equations for the 10×256 system with 16-QAM, operating at $E_b/N_0 = -5$ dB, was 2.41×10^{-5}, which was only 1.55×10^{-6} different from the simulation result. Therefore, the derived PDFs can be efficiently utilized to evaluate the performance of OFDM-MMIMO systems.**

***Index Terms*—OFDM, massive MIMO systems, MRC detection, and Gaussian mixture model**

I. Introduction

For decades, the volume of data traffic on cellular networks has soared due to the increase in the number of user terminals. As a result, the enhancement of cellular networks is regularly required to maintain its quality of service. During the pass few years, the 5-th generation new radio (5G NR) cellular network standard has been developed by a number of researchers, and massive multiple-input, multiple-output (MMIMO) system has become an important technique to eliminate the effects of path loss and improve the spatial multiplexing gain of the system [1]. Since the 5G NR cellular network uses very high frequency bands for data transmission between base station and user terminal, a large number of antennas can be implemented at the base station. As the results, huge gains in robustness, spectral and energy efficiencies can be achieved utilizing spatial multiplexing techniques.

Since the number of antenna arrays at the base station of MMIMO systems is very large, the operational complexity of the receiver becomes an important factor for hardware implementation. As a result, linear symbol detection is recommended for practical systems, since its computational complexity is much less than that of other techniques. Moreover, the Gram matrix of MMIMO system converges into a constant diagonal matrix. Therefore, an excellent performance can be obtained utilizing linear symbol detectors [2].

Maximal-ratio combining (MRC) detection is the lowest-complexity linear symbol detector, and a number of research works has focused on the performance analysis of this detector over the Rayleigh fading channel. The effects of feedback delay and channel estimation errors in MIMO system with MRC detection were investigated in [3]. The authors derived a moment-generating function (MGF) of the output signal-to–interference-plus-noise ratio (SINR) of the system. The MGF was then utilized to analyze the probability distribution function (PDF) and the cumulative distribution function (CDF) of the output SINR, and the symbol error rate of the system. The analytical results from these equations closely matched the simulation results. J. Beiranvand, in [4], derived a PDF of the SINR of a MMIMO system as well as its approximation in forms of Gamma distribution. The authors then used the PDFs to determine the probability of bit error (PBE) and the outage probabilities of the system, utilizing *M*-ary quadrature amplitude modulation (QAM). In 2019, Y. Hama analyzed the PBE of the MMIMO system with MRC detection [5]. The PDF of the output of the MRC detector was derived in the research work, and the authors used the equation to analyze the PBE of the system with binary and quadrature phase shift keying (BPSK, QPSK), and M-QAM.

Recently, we derived in [6] the PDFs of the co-channel interference (CCI) and the effective noise (EN) of MRC detection for an OFDM-MMIMO system that utilizes Gray-coded, BPSK/QPSK and M-QAM modulation. Joint probabilities of random variables in the mathematical expressions of CCI and EN were employed to derive their PDFs. However, a number of correlated variables in the equations were assumed to be independent random variables resulting in small deviations between the analytical results and the actual PDF values. In addition, the distribution of the CCI and the EN of MRC detection is generally approximated as a Gaussian distribution. However, there is no research work regarding a closed-form expression for the PDF and the noise variance. Therefore, a mathematical expression for the Gaussian approximation is still required to simplify the performance analysis.

In this paper, we built upon the work, published in [6], to improve the accuracy of the PDFs of the CCI and the EN of MRC detection for OFDM-MMIMO system. The diagonal elements of the Gram matrix, used in MRC detection, converges to a constant values if the number of the receive antennas is large enough. Therefore, this approximation can be used to simplify the PDFs of CCI and EN for MRC detection. A variable, in the mathematical expression of the CCI and the EN, is approximated as a constant, and

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the PDFs are then derived from the equation. Additionally, we firstly prove that the PDFs of the CCI and the EN in the OFDM-MMIMO system, using Gray-coded BPSK and QPSK modulation, converge to a Gaussian distribution. In addition, if Gray-coded M-QAM is utilized by the system, the PDF tends to be a combination of multivariate Gaussian distributions. As a result, the CCI and the EN of the system can be demonstrated utilizing Gaussian mixture model (GMM). Closed-form expressions of the noise variance for the Gaussian approximation are also derived in this paper. Furthermore, the PDFs and its approximate equations are utilized to evaluate the PBE of the OFDM-MMIMO system.

The remainder of this paper is organized as follows. Section II and III describe the system model of a OFDM-MMIMO system and the effects of CCI and the EN in the MRC detection, respectively. Next, the procedure to derive the PDFs of the CCI and the EN, proposed in [6], is summarized in section IV. We then introduce the proposed method to enhance the accuracy of the PDFs of the CCI and the EN of MRC detection in section V. The asymptotic PDFs of the CCI and the EN in terms of the GMM are then derived in section VI. In section VII, the PBE of OFDM-MMIMO system is analyzed from the derived PDFs. Monte-Carlo simulation is chosen by this research work to evaluate the proposed equations, and the results are discussed in section VIII.

II. System model

Fig. 1 shows a block diagram of an uplink $N_t \times N_r$ OFDM-MMIMO system, where the number of the transmit and the receive antennas are represented as N_t and N_r, respectively. M denotes the number of constellation points of the QAM, and N_f is the number of sub-carriers. The transmitter delivers the information bits $\mathbf{d} \in \mathbb{R}^{N_t \times N_b}$ to the receiver through the MMIMO channel. These information bits are firstly separated into N_t sub-streams, where the size of information bits per sub-stream is denoted as $N_b = \log_2(M)N_f$. The symbol mapper uses $\mathbf{d}$ to produce the transmit symbol matrix, $\mathbf{X} \in \mathbb{C}^{N_t \times N_f}$, in frequency domain. Next, the inverse Fast-Fourier Transform (IFFT) is applied row-wise to transfer the transmit symbols in time domain $\psi \in \mathbb{C}^{N_t \times N_f}$. Cyclic prefix (CP) samples are then added to the transmit symbol to minimize the effects of interblock interference (IBI), and the outcome from this operation is finally transmitted through the MMIMO channel. The receiver uses the received symbol κ^{cp} to estimate the transmit information $\hat{\mathbf{d}}$. Firstly, the CP samples are removed to obtain $\kappa \in \mathbb{C}^{N_r \times N_f}$. The FFT operation is then applied to transfer the received samples in frequency domain and produce $\mathbf{Y} \in \mathbb{C}^{N_r \times N_f}$. Finally the MMIMO detector is utilized to estimate the received information $\hat{\mathbf{d}}$.

The frequency-selective Rayleigh fading channel is considered in this paper. The number of CP is chosen to be larger than the maximum delay spread to eliminate the effects of IBI at the receiver. The received symbols in frequency domain at the f-th sub-carrier can be expressed as

$$\mathbf{Y}_f = \mathbf{H}_f \mathbf{X}_f + \mathbf{W}_f. \tag{1}$$

$\mathbf{H}_f \in \mathbb{C}^{N_r \times N_t}$ in (1) denotes the CFR of the f-th sub-carrier. Focusing on systems with no line-of-sight links between the transmitter and the receiver, each component $H_{n,m}$ of $\mathbf{H}_f$ is assumed to be a zero-mean, complex-valued, Gaussian distribution $\mathcal{CN}(0, 2\sigma_h^2)$. The variance per dimension σ_h^2 is fixed at 0.5 to normalize the path gain. $\mathbf{W}_f \in \mathbb{C}^{N_r \times 1}$ denotes the matrix of noise vector for the f-th sub-carrier. Each element W_n in $\mathbf{W}_f$ exhibits a $\mathcal{CN}(0, 2\sigma_w^2)$. Let E_s and γ_b denote the average symbol energy and the average signal-to-noise ratio (SNR) per bit, respectively, the value of σ_w^2 is given by

$$\sigma_w^2 = \frac{N_t E_s}{2\log_2(M)\gamma_b}. \tag{2}$$

III. The CCI and the EN of MRC detection

This section summarizes the operation of MRC detection and the mathematical expression of CCI and EN on MRC detection for OFDM-MMIMO systems. Generally, the MRC receiver estimates the transmitted symbol $\hat{\mathbf{X}}_f$ as

$$\tilde{\mathbf{X}}_f = \mathbf{H}_f^{\dagger} \mathbf{Y}_f, \tag{3}$$

where the Hermitian transpose of $\mathbf{H}_f$ is represented as $\mathbf{H}_f^{\dagger}$. By substituting $\mathbf{Y}_f$ from (1) in (3), $\tilde{\mathbf{X}}_f$ can be rewritten as

$$\tilde{\mathbf{X}}_f = \mathbf{G}_f \mathbf{X}_f + \mathbf{H}_f^{\dagger} \mathbf{W}_f. \tag{4}$$

The value of the first term in (4) depends on $\mathbf{X}_f$, however, it is scaled by the Gram matrix $\mathbf{G}_f = \mathbf{H}_f^{\dagger}\mathbf{H}_f \in \mathbb{C}^{N_t \times N_t}$. Therefore, each m-th element $\tilde{X}_m$ of $\tilde{\mathbf{X}}_f$ is then normalized by the m-th diagonal component of the Gram matrix $G_{m,m} = \sum_{n=1}^{N_r} |H_{n,m}|^2$ to produce the estimated transmitted symbol $\hat{X}_m$, i.e.,

$$\hat{X}_m = \frac{\tilde{X}_m}{G_{m,m}}. \tag{5}$$

The metric $\hat{X}_m$, obtained by the operation of MRC detection, is usually contaminated by the effects of the CCI and EN, caused by the MMIMO channel. By substituting $\tilde{X}_m$ from (4) in (5), $\hat{X}_m$ becomes [6]

$$\hat{X}_m = X_m + \frac{\sum_{k=1, k\neq m}^{N_t} G_{m,k} X_k}{G_{m,m}} + \frac{\sum_{n=1}^{N_r} H_{n,m}^* W_n}{G_{m,m}}. \tag{6}$$

IV. PDF of CCI and EN of MRC detection [6]

The second and the third terms in (6) demonstrate the CCI and the EN of MRC detection, respectively. We use Z_m to express these terms, i.e.,

$$Z_m = \frac{\sum_{k=1, k\neq m}^{N_t} G_{m,k} X_k}{G_{m,m}} + \frac{\sum_{n=1}^{N_r} H_{n,m}^* W_n}{G_{m,m}}. \tag{7}$$

Let $\{Z_m^{\lambda}\}_{\lambda=\{\mathrm{I,Q}\}}$ denotes the in-phase and the quadrature component of Z_m. The PDF of Z_m^{λ} was proposed in [6] and the procedure for deriving the PDF is now briefly explained in this section. If $\alpha_{n,m} = \sum_{k=1, k\neq m}^{N_t} H_{n,k} X_k$ and $\beta_{n,m} = \alpha_{n,m} + W_n$, Z_m from (7) can be rewritten as

$$Z_m = \frac{\sum_{n=1}^{N_r} H_{n,m}^* \beta_{n,m}}{G_{m,m}}, m = 1, 2, \ldots, N_t. \tag{8}$$

If $\eta_m = \sum_{n=1}^{N_r} H_{n,m}^* \beta_{n,m}$, Z_m in (8) becomes

$$Z_m = \frac{\eta_m}{G_{m,m}}. \tag{9}$$

Since η_m and $G_{m,m}$ in (9) are produced from functions of $H_{n,m}$ and W_n, which are random variables, we used their

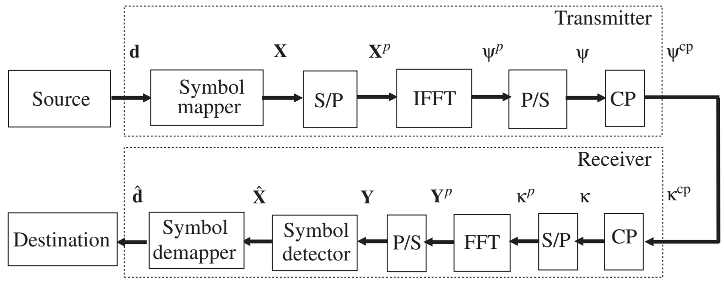

Fig. 1. Block diagram of OFDM-MMIMO system

joint probabilities to derive the PDF of the variables in [6]. As a result, the PDF of Z_m^λ for M-QAM is expressed as

$$p_z^{M-\mathrm{QAM}}(Z_m^\lambda) = \frac{(2N_t-2)!}{\Gamma(N_r)^2\Delta^{2N_t-2}} \times \sum_{\sum_{v=1}^{\Delta} k_v=(2N_t-2)} \frac{\sigma_{\beta,\{k\}}^{N_r}}{\prod_{v=1}^{\Delta} k_v!} \times \sum_{i=1}^{N_r} \frac{\sigma_h^{N_r-i+1}(N_r+i-2)!(2N_r-i)!|Z_m^\lambda|^{N_r-i}}{2^{2i-2}\Gamma(i)(N_r-i)!(\sigma_{\beta,\{k\}}+2\sigma_h|Z_m^\lambda|)^{2N_r-i+1}}, \tag{10}$$

where $\sigma_{\beta,\{k\}}^2$ is given as

$$\sigma_{\beta,\{k\}}^2 = \sigma_h^2 \sum_{v=1}^{\Delta} k_v (\bar{X}_v^\lambda)^2 + \sigma_w^2. \tag{11}$$

Δ denotes the size of constellation points per dimension, and $\{\bar{X}_v^\lambda\}_{v=1}^{\Delta}$ represents the constellation points per dimension, i.e., $\{-3,-1,1,3\}$ for 16-QAM. $\Gamma(K)$ is the gamma function of variable K. Additionally, it was shown for BPSK and QPSK that the $p_z(Z_m^\lambda)$ is given as

$$p_z^{\mathrm{BPSK/QPSK}}(Z_m^\lambda) = \frac{\sigma_\beta^{N_r}}{\Gamma(N_r)^2} \times \sum_{i=1}^{N_r} \frac{\sigma_h^{N_r-i+1}(N_r+i-2)!(2N_r-i)!|Z_m^\lambda|^{N_r-i}}{2^{2i-2}\Gamma(i)(N_r-i)!(\sigma_\beta+2\sigma_h|Z_m^\lambda|)^{2N_r-i+1}}, \tag{12}$$

where the variance σ_β^2 is defined as

$$\sigma_{\beta,\mathrm{BPSK}}^2 = (N_t-1)\sigma_h^2 + \sigma_w^2, \tag{13a}$$
$$\sigma_{\beta,\mathrm{QPSK}}^2 = 2(N_t-1)\sigma_h^2 + \sigma_w^2. \tag{13b}$$

V. Improved PDF of Z_m

η_m and $G_{m,m}$ in (9) are dependent random variables, since they are produced from $H_{n,m}$. As a result, there is still a small deviation between the true value of the PDF and the computation from the derived equations in (10), (12). We now utilize an approximation to solve the issue regarding the dependent variables in (9). It is well known that the Gram matrix of a OFDM-MMIMO systems converge to a constant diagonal matrix, and its diagonal component $G_{m,m}$ can be approximated as a constant Ω, i.e., [5]

$$\Omega = 2N_r\sigma_h^2. \tag{14}$$

By substituting Ω from (14) into $G_{m,m}$ of (9), Z_m becomes

$$Z_m \simeq \frac{\eta_m}{\Omega}. \tag{15}$$

Since Z_m in (15) is the ratio of η_m and the constant Ω, $p_z(Z_m^\lambda)$ can be derived in terms of $p_\eta(\eta_m^\lambda)$ by utilizing (4.19) in [7] as

$$p_z(Z_m^\lambda) = \Omega p_\eta(\Omega Z_m^\lambda). \tag{16}$$

The PDF of η_m of the OFDM-MMIMO system, utilizing M-QAM was previously derived in [6] as

$$p_\eta^{M-\mathrm{QAM}}(\eta_m^\lambda) = \frac{(2N_t-2)!}{\Gamma(N_r)\Delta^{2N_t-2}} \times \sum_{\sum_{v=1}^{\Delta} k_v=2N_t-2} \frac{e^{-\frac{|\eta_m^\lambda|}{\sigma_h\sigma_{\beta,\{k\}}}}}{\prod_{v=1}^{\Delta} k_v!} \times \sum_{i=1}^{N_r} \frac{(N_r+i-2)!}{2^{N_r+i-1}\sigma_h\sigma_{\beta,\{k\}}\Gamma(i)(N_r-i)!} \left(\frac{|\eta_m^\lambda|}{\sigma_h\sigma_{\beta,\{k\}}}\right)^{N_r-i}. \tag{17}$$

$\{k_v\}_{v=1}^{\Delta}$ in (17) denotes an integer and its value varies from 0 to $(2N_t-2)$. If $p_\eta(\eta_m^\lambda)$ in (17) is substituted in (16), the PDF of Z_m^λ becomes

$$p_z^{M-\mathrm{QAM}}(Z_m^\lambda) = \frac{(2N_t-2)!}{\Delta^{2N_t-2}} \times \sum_{\sum_{v=1}^{\Delta} k_v=2N_t-2} \frac{f_1(Z_m^\lambda, \sigma_{\beta,\{k\}})}{\prod_{v=1}^{\Delta} k_v!}, \tag{18}$$

where

$$f_1(Z_m^\lambda, \sigma_\beta) = \frac{1}{\Gamma(N_r)} \exp\left(-\frac{2N_r\sigma_h|Z_m^\lambda|}{\sigma_\beta}\right) \times \sum_{i=1}^{N_r} \frac{(N_r+i-2)!|Z_m^\lambda|^{N_r-i}}{2^{2i-2}\Gamma(i)(N_r-i)!} \left(\frac{N_r\sigma_h}{\sigma_\beta}\right)^{N_r-i+1}. \tag{19}$$

It is worth noting that the PDFs of Z_m^λ of the OFDM-MMIMO system with BPSK and QPSK modulation can be derived by utilizing (16) and the PDF of η_m from [6], i.e.,

$$p_\eta^{\mathrm{BPSK/QPSK}}(\eta_m^\lambda) = \frac{e^{-\frac{|\eta_i^\lambda|}{\sigma_h\sigma_\beta}}}{\sigma_h\sigma_\beta\Gamma(N_r)} \times \sum_{i=1}^{N_r} \frac{(N_r+i-2)!}{2^{N_r+i-1}\Gamma(i)(N_r-i)!} \left(\frac{|\eta_m^\lambda|}{\sigma_h\sigma_\beta}\right)^{N_r-i}, \tag{20}$$

resulting in

$$p_z^{\mathrm{BPSK/QPSK}}(Z_m^\lambda) = f_1(Z_m^\lambda, \sigma_\beta). \tag{21}$$

VI. Approximate PDF of Z_m^λ

Evidently, the computation of $p_z(Z_m^\lambda)$ using (18) and (21) requires several arithmetic operations to determine its value, and its computational complexity grows exponentially according to the value of N_t and Δ. As a result, the proposed equations are still impractical for the OFDM-MMIMO system, where a high-order modulation scheme is utilized. Therefore, the approximate PDFs of CCI and EN are introduced in this section. Focusing on Z_m in (9), η_m^λ is a $2N_r$ summations of the product of $H_{n,m}^{\lambda,*}$ and $\beta_{n,m}^\lambda$. $H_{n,m}^*$ is a Gaussian random variable, and $p_\beta(\beta_{n,m}^\lambda)$ in [6] is also the sum of Gaussian distributions, i.e.,

$$p_\beta^{M-\mathrm{QAM}}(\beta_{n,m}^\lambda) = \frac{(2N_t-2)!}{\Delta^{2N_t-2}} \times \sum_{\sum_{v=1}^{\Delta} k_v = 2N_t-2} \frac{f_g(\beta_{n,m}^\lambda, \sigma_{\beta,\{k\}})}{\prod_{v=1}^{\Delta} k_v!}. \tag{22}$$

$f_g(\beta_{n,m}^\lambda, \sigma_{\beta,\{k\}})$ is a zero-mean, Gaussian distribution, which is defined as

$$f_g(\beta_{n,m}^\lambda, \sigma_{\beta,\{k\}}) = \frac{1}{\sqrt{2\pi\sigma_{\beta,\{k\}}^2}} \exp\left(-\frac{(\beta_{n,m}^\lambda)^2}{2\sigma_{\beta,\{k\}}^2}\right). \tag{23}$$

Therefore, the PDF of η_m^λ of the OFDM-MMIMO system, utilizing M-QAM can be approximated as a sum of the Gaussian distribution, i.e.,

$$p_\eta^{M-\mathrm{QAM}}(\eta_m^\lambda) \simeq \frac{(2N_t-2)!}{\Delta^{2N_t-2}} \sum_{\sum_{v=1}^{\Delta} k_v = 2N_t-2} \frac{f_g(\eta_m^\lambda, \sigma_{\eta,\{k\}})}{\prod_{v=1}^{\Delta} k_v!}. \tag{24}$$

The variance $\sigma_{\eta,\{k\}}^2$ in (24) depends on the value of $k_1, k_2, \dots k_\Delta$, and is determined by $2N_r\sigma_h^2\sigma_{\beta,\{k\}}^2$, resulting in

$$\sigma_{\eta,\{k\}}^2 = 2N_r\left(\sigma_h^4 \sum_{v=1}^{\Delta} k_v(\bar{X}_v^\lambda)^2 + \sigma_h^2\sigma_w^2\right). \tag{25}$$

If the asymptotic PDF of η_m^λ in (24) is substituted into (16), the PDF of Z_m^λ can be approximated as

$$p_z^{M-\mathrm{QAM}}(Z_m^\lambda) \simeq \frac{(2N_t-2)!}{\Delta^{2N_t-2}} \times \sum_{\sum_{v=1}^{\Delta} k_v = 2N_t-2} \frac{f_g(Z_m^\lambda, \sigma_{z,\{k\}})}{\prod_{v=1}^{\Delta} k_v!}. \tag{26}$$

By utilizing the relationship between the PDF of η_m^λ and Z_m^λ from (16), $\sigma_{z,\{k\}}^2$ in (26) is expressed as $\sigma_{\eta,\{k\}}^2/\Omega^2$, i.e.,

$$\sigma_{z,\{k\}}^2 = \frac{\sigma_h^2 \sum_{v=1}^{\Delta} k_v(\bar{X}_v^\lambda)^2 + \sigma_w^2}{2N_r\sigma_h^2}. \tag{27}$$

The PDF in (26) is the GMM, which can be used to approximate $p_z(Z_m^\lambda)$ in the OFDM-MMIMO system, employing M-QAM. In addition, the PDF of Z_m^λ of the OFDM-MMIMO system that utilizes BPSK and QPSK modulation can be derived by using the Gaussian approximation. Since the PDF of $\beta_{n,m}^\lambda$ for the system in [6] is a Gaussian distribution with the variance σ_β^2 in (13), the PDF of Z_m^λ becomes a Gaussian distribution, i.e.,

$$p_z^{\mathrm{BPSK/QPSK}}(Z_m^\lambda) \simeq f_g(Z_m^\lambda, \sigma_z), \tag{28}$$

where the variance σ_z^2 in (28) is determined by $2N_r\sigma_h^2\sigma_\beta^2/\Omega^2$, resulting in

$$\sigma_{z,\mathrm{BPSK}}^2 = \frac{\sigma_h^2(N_t-1) + \sigma_w^2}{2N_r\sigma_h^2}, \tag{29a}$$

$$\sigma_{z,\mathrm{QPSK}}^2 = \frac{2\sigma_h^2(N_t-1) + \sigma_w^2}{2N_r\sigma_h^2}. \tag{29b}$$

VII. PBE ANALYSIS

In this section, we employ $p_z(Z_m^\lambda)$ and their approximations in (18), (21), (26), and (28) to determine the PBE performance of the OFDM-MMIMO system. Focusing on the system that utilizes BPSK and QPSK modulation with the constellation points per dimension $\{-1, 1\}$ and $\Delta = 2$, the pairwise error probability (PEP) P_s of the system can be evaluated from $p_z(Z_m^\lambda)$ as [6]

$$P_s = \int_0^\infty p_z(Z_m^\lambda + 1) dZ_m^\lambda. \tag{30}$$

If the $p_z(Z_m^\lambda)$ in (21) is substituted in (30), the PEP of the system utilizing BPSK and QPSK modulation, becomes

$$P_s^{\mathrm{BPSK/QPSK}} = f_2(\sigma_\beta), \tag{31}$$

where

$$f_2(\sigma_\beta) = \frac{e^{-\frac{2N_r\sigma_h}{\sigma_\beta}}}{\Gamma(N_r)} \times \sum_{i=1}^{N_r} \frac{(N_r+i-2)!}{\Gamma(i)} \sum_{p=0}^{N_r-i} \frac{1}{2^{N_r+i-p-1}p!}\left(\frac{N_r\sigma_h}{\sigma_\beta}\right)^p. \tag{32}$$

Since the number of constellation points per dimension of the system with BPSK and QPSK modulation is 2, P_s in (31) can be also used to express the PBE P_e of the system. The PEP of the system that utilizes Gray-coded M-QAM is determined by substituting $p_z(Z_m^\lambda)$ from (18) in (30), and the result from this operation is

$$P_s^{M-\mathrm{QAM}} = \frac{(2N_t-2)!}{\Delta^{2N_t-2}} \sum_{\sum_{v=1}^{\Delta} k_v = 2N_t-2} \frac{f_2(\sigma_{\beta,\{k\}})}{\prod_{v=1}^{\Delta} k_v!}. \tag{33}$$

Focusing on the OFDM-MIMO system that uses Gray-coded 16-QAM with constellation points per dimension $\{-3, -1, 1, 3\}$, the PBE of the system can be evaluated from P_s. Due to the fact that the number of constellation points per dimension for real and imaginary part of 16-QAM is 4 and the number of error events per dimension is 6, the PBE of the system can be expressed as [6]

$$P_e^{16-\mathrm{QAM}} = \frac{6P_s^{16-\mathrm{QAM}}}{4\log_2(\Delta)}. \tag{34}$$

By substituting P_s from (33) and $\Delta = 4$ into (34), the PBE of the system with 16-QAM becomes

$$P_e^{16-\mathrm{QAM}} = \frac{3(2N_t-2)!}{2^{4N_t-2}} \sum_{\sum_{v=1}^{4} k_v = 2N_t-2} \frac{f_2(\sigma_{\beta,\{k\}})}{\prod_{v=1}^{4} k_v!}. \tag{35}$$

In addition, if the approximate PDF in (26) and (28) is utilized in (30), P_s for OFDM-MMIMO system with BPSK and QPSK modulation can be approximated as

$$P_e^{\mathrm{BPSK/QPSK}} \simeq Q\left(\frac{1}{\sigma_z}\right). \tag{36}$$

Furthermore, by substituting the approximate PDF in (26) into (30) and (34), P_e for the system with 16-QAM becomes

$$P_e^{16-\mathrm{QAM}}(Z_m^\lambda) \simeq \frac{3(2N_t-2)!}{2^{(4N_t-2)}} \sum_{\sum_{v=1}^{4} k_v = 2N_t-2} \frac{Q\left(\frac{1}{\sigma_{z,\{k\}}}\right)}{\prod_{v=1}^{4} k_v!}. \tag{37}$$

VIII. Simulation results

In this paper, the analytical results from the proposed equations are evaluated using Monte-Carlo simulations. The operation of the $N_t \times N_r$ OFDM-MMIMO system, which was chosen for the simulation, is described in Fig. 1. A frequency-selective Rayleigh fading channel is considered in this paper, and the channel model is as described in section II. QPSK modulation and 16-QAM are chosen for the simulation.

Fig. 2 shows the PDFs of Z_m^I of OFDM-MMIMO system, utilizing 16-QAM and operating at $E_b/N_0 = 0$ dB. $N_t = 10$, and $N_r = \{16, 64, 256\}$. The analytical results from (18) and the approximate PDF in (26) are now compared to that of the simulation. Undoubtedly, the results from the proposed PDFs significantly matched the simulation results. Considering the 10×256 system, the analytical result from the exact PDF at $Z_m^I = 0.5$ was 5.85×10^{-2}, which was only 1.7×10^{-5} different from the simulation result. In addition, the accuracy of the approximate PDF was slightly less than that of the exact PDF, and the deviation between the result from the approximate PDF and the simulation at $Z_m^I = 0.5$ was 5.28×10^{-5}. The analytical results and the simulation results were also compared through the 2-sample Kolmogorov–Smirnov test at the significance level of 5%, and the results confirmed that there is no significant difference between the PDF from the analytical and the simulation results. As expected, the approximation in (14) for small N_r slightly deviates from the simulation results, e.g., for the 10×16 systems at $Z_m^I = 0.5$ was 3.69×10^{-3}.

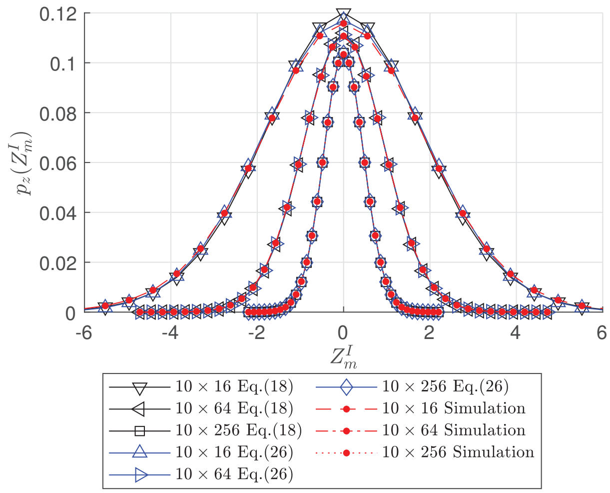

Fig. 2. The PDF of the CCI and the EN of MRC detection

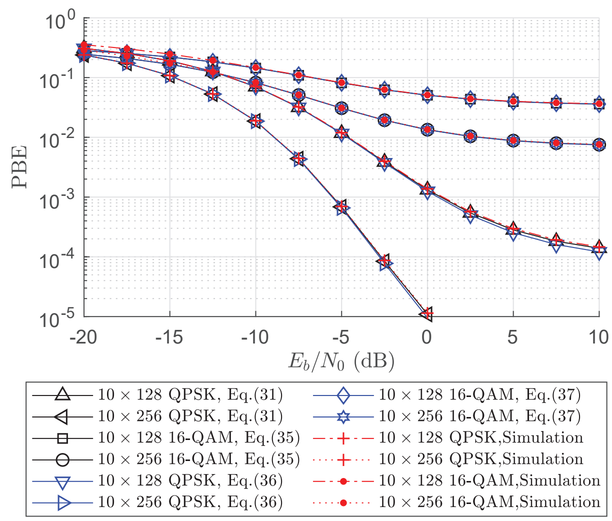

Fig. 3. The PBE of OFDM-MMIMO system

The analytical and the simulation results for the PBE of the OFDM-MMIMO system are illustrated in Fig. 3. The exact PBE analysis in (31) and (35) as well as its approximate PBE in (36) and (37) are also included in the results. QPSK modulation and 16-QAM are chosen for the simulation. These results confirmed that the exact PBE analysis produced an accurate result compared to the simulation. Focusing on the 10×256 system that utilizes 16-QAM, the PBE from the exact equation and the simulation at $E_b/N_0 = 0$ dB was 1.34×10^{-2} and 1.35×10^{-2}, respectively. The difference in the PBE between the analytical and the simulation result was only 1.13×10^{-4}. It is worth noting that the accuracy of the PBE analysis from the approximate equation was slightly less that the number from the exact equations, and the difference between the analytical result at $E_b/N_0 = 0$ dB and the simulation result was 1.65×10^{-4}.

Fig. 4 compare the PBE analysis from the proposed equations with that of Eq. (33) and (36) in [6]. The analytical results from Eq. (28) and (35) in [5] are also included in the results. Both QPSK modulation and 16-QAM are chosen for the comparison. The results confirmed that the proposed equations for the system with 16-QAM outperform its analytical results. The PBE from the exact PBE analysis at $E_b/N_0 = -5$ dB was 9.7×10^{-3}, and the difference between this analytical results with that of the simulation was 1.27×10^{-4}. The deviation between the analytical result from the Eq. (35) in [5] and the Eq. (36) in [6] was 1.8×10^{-3}, and 5.36×10^{-4}, respectively. The proposed exact PBE analysis for the system with QPSK modulation also produced more accurate results than that of the analytical result from the derived equation in [6]. The PBE from the proposed equation for the system with QPSK modulation, operating at $E_b/N_0 = -5$ dB was 2.41×10^{-5}, and the difference between this analytical result and the simulation result was 1.55×10^{-6}. The difference in PBE between the analytical result from the equation in [6] and the simulation result was 1.91×10^{-5}. However, the PBE analysis for the system wih QPSK modulation from the Eq. in [5] was slightly closer to the simulation result than that of the proposed equation. The deviation between the simulation results and the outcome from the equation in [5] at $E_b/N_0 = -5$ was 2.52×10^{-5} , i.e., only 4.16×10^{-7} different from the simulation result.

IX. Conclusion

This research work improves the accuracy of the PDFs of CCI and EN of MRC detection for the OFDM-MMIMO system proposed in [6]. We have used mathematical expressions to demonstrate the effects of CCI and EN, and derived an approximation to analyze the PDFs. We then proved that the CCI and EN can be approximated as a GMM and their PDFs tends to be a combination of multivariate Gaussian distribution. The derived PDFs are then utilized to evaluate the PBE of OFDM-MMIMO system. According to the simulation and the analytical results, the proposed equation produced an accurate PDF and PBE, especially if N_r is large enough. In addition, the computational complexity of

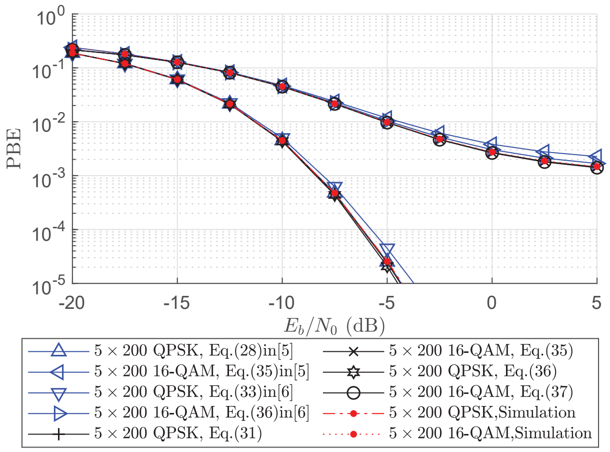

Fig. 4. Comparison of the PBE from the derived equations

the approximate equations is considerably less than that of the exact equation, whereas the accuracy of the analytical result is slightly reduced. However, the proposed equations still cannot provide accurate result for the system, where N_r is too small, since we use approximations in this research work to derive the equations. Therefore, a more accurate PDF of the CCI and the EN of MRC detection is still required.

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