Generalised Channel State Information Outdatedness Modelling Using MRC

Khoa N. Le[a] and Vo Nguyen Quoc Bao[b]

***Abstract*—This paper offers unified modelling for opportunistic-relay outdatedness using the joint distribution of bivariate exponential-correlated generalised-Rician fading. Line-of-sight power and larger-than-two degrees of freedom under independently-and-identically-distributed Rayleigh fading environments employing maximal ratio combining (MRC) are considered. Unavoidable infinite summations are employed, which are shown to fast-converge for a finite number of terms. Ergodic channel capacity is employed for performance assessment. Thorough comparisons between performance of selection combining and MRC are also given. Monte Carlo simulation is employed to validate this work.**

I. Introduction

Relay networks have attracted attention from researchers in recent years as reported in [1–5] because of their versatility and robustness. Simultaneously being useful, opportunistic relays can experience outdatedness—typically caused by external interferences—which also requires research efforts to gain thorough understanding.

A. Existing literature

Relay outdatedness has been studied in the literature as shown in [6–11], from which it appears that relay outdatedness has (i) been modelled under non-line-of-sight (NLoS) conditions with two degrees of freedom (DoF) or Rayleigh fading using the joint probability density function (pdf) of bivariate exponential-correlated (exp.c.) Rayleigh fading [12]. In addition, a conditional pdf of bivariate exp.c. Nakagami-m fading has been reported in [13], which has inspired the findings reported in [6], (ii) not been modelled under line-of-sight (LoS) conditions.

It is important to understand that theoretical developments for outdated-channel-state-information (oCSI) relays are based on theories of correlated fading, where $0 \leq \epsilon = \rho^2 < 1$ has been defined as the channel coherence coefficient and ρ is channel correlation coefficient. Research on oCSI relays has been started with the work of [6] under independent-and-identically-distributed (i.i.d.) Rayleigh fading with selection combining (SC) deployment using the earlier results given in [13] for the conditional pdf of bivariate exp.c. Nakagami-m fading. Logically, relay outdatedness employing the joint pdfs of bivariate exp.c. (i) Nakagami-m with $2m$ DoF and zero-LoS power, (ii) Rician with two-DoF LoS power and (iii) generalised-Rician fading with DoF and LoS power, can be performed to widen practical applications of relay networks and relay outdatedness under generalised conditions. However, this progress has not yet been made, which is partly because of (i) the limitation of the joint pdfs of bivariate exp.c. Rician and generalised-Rician fading, (ii) the impracticality of the joint pdf of bivariate exp.c. Rician fading with unavoidable four nested but converged infinite summations, (iii) mathematical intractability which has typically and notoriously occurred under correlated fading environments. It is also noted that the generalised-Rician fading is an advanced fading environment, which reduces to Nakagami-m fading for $s_{n_1}^2 = 0, n_1 \geq 2$ and Rician fading for $n_1 = 2, s_2^2 = K \geq 0$. However, analyses under generalised-Rician fading have been limited in the existing literature because of its comprehensiveness and simultaneously notorious intractability.

It has appeared that the theory of correlated fading has chronically relied on SC to advance, not maximal ratio combining (MRC), which is mainly because of intractability. Specifically, the number of variates represents the diversity order. This means that increasing the diversity order increases the number of joint variates. On the other hand, the deployment of diversity techniques for relays aims to combat fading severity, of which Rayleigh fading has been commonly employed as relay operating environment for simplification purposes. This means that increasing the diversity order lowers the negative impact of fading severity on the relay system. The roles of diversity for the theory of correlated fading and for combating fading severity are thus mathematically different.

As technologies advance, so do relay networks with possible deployment of flying drones and opportunistic smart mobile devices. Even though progress has been made for wireless oCSI relay networks, the theory of relay outdatedness has long remained to be valid for two zero-mean Gaussian random variables (RVs) under Rayleigh fading, i.e. two DoF. In addition, as shown in [8–11], oCSI relays have found wide applications in different branches of wireless communications, which suggests that comprehensive relay outdatedness modelling is fundamentally necessary.

B. Contribution

The distinctive contribution of this paper is the mathematical modelling for relay outdatedness using larger-than-two DoF and LoS power. The key difference of this work from other works is the deployment of the bivariate exp.c. generalised-Rician fading for unified relay-outdatedness modelling, where the exp.c. model defines elements of the underlying covariance matrix as $m_{ij} = \sigma^2 \rho^{|i-j|}$. Two objectives are designed to achieve this unique aim (i) deriving the conditional pdf of outdated relay instantaneous signal-to-noise ratio (SNR) given the instantaneous non-outdated relay SNR and (ii) obtaining the pdf of Rayleigh

[a]School of Engineering Design and Built Environment, Western Sydney University, Australia. Email: lenkhoa@gmail.com

[b]Posts and Telecommunications Institute of Technology, Ho Chi Minh City, Vietnam. Email: baovnq@ptithcm.edu.vn

fading under LoS-and-n-DoF-oCSI conditions—referred to as "unified-oCSI" conditions in this paper—which provides comprehensiveness to the proposed findings. To the best of the author's knowledge, unified-oCSI conditions have not been performed in the literature, except a relay's oCSI analysis under n DoF has been reported in [13].

C. Organisation

This paper is organised as follows. Section II describes the relay network under study. Section III obtains the pdf of Rayleigh fading under unified-oCSI conditions employing MRC. Section IV studies the ergodic channel capacity (ECC) under different scenarios so that additional insight can be gained. Trade-off between different parameters can be identified. Section V concludes the main results and outlines possible future work.

Notation: r_1, r_2 are the magnitudes of the n-dimensional envelopes comprising of $n \geq 3$ underlying Gaussian random variables (RVs), σ^2 is the common variance of the Gaussian RVs, $\mu_i, i = 1, \ldots, n$ is the mean of the ith Gaussian RV, $s_n^2 = \sum_{i=1}^{n} \mu_i^2 \geq 0$ is the non-centrality parameter for n DoF, $0 \leq \rho < 1$ is the channel correlation coefficient and $m \geq 1.5$.

II. System Model

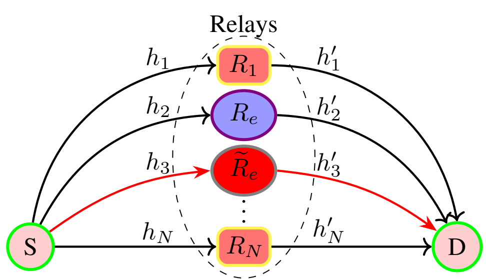

Fig. 1. A unified-oCSI relay network employing MRC

Fig. 1 schematically describes a bank of decode-and-forward (DF) opportunistic relays (ORs), which operate under (i) unified-oCSI conditions and (ii) MRC deployment, which means that all N relays are included for transmission and their individual channel fading gains under i.i.d. Rayleigh fading are coherently summed at the Destination. Generally, relay outdatedness is caused by external disturbances, which means that the non-outdated relay R_e is typically not the chosen relay at the instance of transmission, but its outdated version $\widetilde{R}_e$ is selected instead.

Typically, the joint pdf of bivariate exp.c. Rayleigh fading has been employed to model fading severity as shown in [6], which limits their analyses under 2-DoF and zero-LoS-power conditions. In this paper, relay outdatedness is mathematically modelled using both LoS power K and DoF n. This means that (i) the outdated relay $\widetilde{R}_e$ SNR γ_t and the non-outdated relay R_e SNR γ are mathematically described by the joint pdf of bivariate exp.c. generalised-Rician fading, which generalises existing findings given in [6] and (ii) the impact of generalised-Rician fading severity is transformed into that of relay outdatedness under DoF and LoS power. The Destination D communicates with relays using a broadcasting protocol (i) D polls for active relays, (ii) relays respond to D and they exchange their CSI. During this phase, Source's CSI is also broadcast to relays, (iii) D consolidates the list of active users and computes the coherent sum of individual relay channel fading gains using its MRC receiver. Assuming that messages can be successfully decoded, outage occurs if D could not locate a sufficient number of active relays $N_S \leq N$. In this case, D keeps polling until N_S relays have voluntarily agreed to participate in the transmission. For relay outdatedness, it is noted that the concept of (i) "LoS power" is directly related to relay's CSI correlation, which is described by ϵ and (ii) "DoF" implies that relay's CSI can be mathematically constituted using a set of n underlying non-zero-mean equal-variance Gaussian RVs. It is noted that relay's CSI modelling is independent from the fading environment, which is Rayleigh in this paper. As such, using the distribution given in Theorem 1 gives additional insight into how oCSI severity affects wireless system performance under both DoF n and LoS power s_n^2. It is also assumed that (i) the relays operate under i.i.d. Rayleigh fading and (ii) an end-to-end analysis is not offered in this paper to ensure that mathematical tractability is successfully retained. Given that unified relay outdatedness and non-outdatedness are two RVs, which can be described as random envelopes of generalised-Rician fading. A (k, j)th equal-correlated (equ.c.) generalised-Rician envelope can be described as

$$Q_{kj} = \sqrt{1-\rho}X_{kj} + \sqrt{\rho}X_{0j} + \mu_{1j} + \sqrt{1-\rho}Y_{kj} + \sqrt{\rho}Y_{0j} + \mu_{2j}, \qquad (1)$$

where $X_{0j}, X_{kj}, Y_{0j}, Y_{kj}$ are independent underlying Gaussian RVs, μ_{1j}, μ_{2j} are means of the real and imaginary parts of the jth LoS component, whose power is given by s_n^2. The (l, j)th equ.c. generalised-Rician envelope Q_{lj} can be readily obtained by replacing k with l under (1). From Fig. 1, given the transmit signal x from S, the respective received signals at $R_1, R_e, \widetilde{R}_e, \ldots, R_N$ are $y_i = \sqrt{P}h_i x + n_i, i \in [1, N]$, where n_i is the added white Gaussian noise (AWGN) at ith relay, P is the transmit power, h_i, h'_i are channel fading gains of the ith relay from S and of the ith relay to D respectively. The coherent received signal at D using an MRC receiver is thus $y_D = \sum_{i=1}^{N} \sqrt{P}h'_i\widehat{x} + n_D$, where $\widehat{x}$ is the encoded version of x at relay R_i, n_D is the AWGN at D. For DF relays, it is expected that $h_i \approx h'_i$.

III. The pdf of Rayleigh fading under unified-oCSI conditions employing MRC

Theorem 1: The pdf of Rayleigh fading environment under unified-oCSI conditions employing MRC is given by

$$f(\gamma) = \sum_{k,k_1=0}^{\infty} \frac{\mathcal{S}\gamma^{\frac{\varpi-m+1}{2}} I_{\varpi}\left(2\delta_2\sqrt{\gamma}\right) e^{-\varrho\epsilon\Gamma_a\gamma} {}_1F_1\left(\mathcal{A}; A\gamma\right)}{I_{m-1}\left(2\delta_4\sqrt{\gamma}\right)}, \qquad (2)$$

$$\mathcal{S} = \frac{\mathcal{Y}\delta_1^{\varpi}\delta_3^{\varpi+2k_1}\Gamma(N+k_1+\varpi)}{\Sigma_2^{N+k_1+\varpi}k_1!\Gamma(\varpi+1)\Gamma(k_1+\varpi+1)}, \qquad (3)$$

$$\Sigma_1 = \frac{1}{\Sigma}, \Sigma_2 = \Sigma_1 + \Gamma_t\varrho, \mathcal{Y} = \frac{\Delta\Delta_1\Upsilon^{\frac{m-1}{2}}s_n^{m-1}e^{\xi s_n^2}}{4\Sigma^N\Upsilon_t\xi\Gamma(N)}, \qquad (4)$$

$$\delta_1 = \frac{\xi\varrho\sqrt{\epsilon}}{\sqrt{\Upsilon\Upsilon_t}}, \delta_2 = \frac{\xi\sqrt{\varrho_1 s_n^2}}{\sqrt{\Upsilon}}, \delta_3 = \frac{\xi\sqrt{\varrho_1 s_n^2}}{\sqrt{\Upsilon_t}}, \qquad (5)$$

where $A = \frac{\delta_1^2}{\Sigma_2}, \Delta = \frac{4\Gamma(m-1)}{\xi^{m-3}(1-\epsilon)}\left(\frac{(1+\sqrt{\epsilon})^2}{s_n^2\sqrt{\epsilon}}\right)^{m-1} e^{-\frac{2\xi s_n^2}{1+\sqrt{\epsilon}}}$, $\Delta_1 = \varpi(-1)^k \binom{n+k-3}{n-3}, \delta_4 = \frac{\xi s_n}{\sqrt{\Upsilon}}$, $\varpi = m+k-1$, $\xi = \frac{\Gamma(m+1){}_1F_1\left(m+1;m;s_n^2\right)e^{-s_n^2}}{\Gamma(m)}$, $\varrho = \frac{1}{1-\epsilon}, \varrho_1 = \frac{1}{(1+\sqrt{\epsilon})^2}$, $\Gamma_a = \frac{\xi}{\Upsilon}, \Gamma_t = \frac{\xi}{\Upsilon_t}, \mathcal{A} = \{N + k_1 + \varpi; \varpi + 1\}$, $0 \leq \epsilon < 1$ is the system coherence coefficient, N is the number of relays, Σ is the average SNR over Rayleigh fading employing MRC, Υ, Υ_t are the average SNRs of the instantaneous SNRs γ, γ_t of the non-outdated relay R_e and the outdated relay $\hat{R}_e$ respectively, $n = 2m$ is the DoF, ξ is the second moment of generalised-Rician distribution, ${}_1F_1(a;b;c)$ is Kummer confluent hypergeometric function, $I_v(x)$ is the vth-order modified Bessel function of the first kind and $\Gamma(p) = \int_0^\infty t^{p-1}e^{-t}dt, p > 0$ is Gamma function.

Proof: Please see Appendix. ∎

Remark 1: To the best of the author's knowledge, Theorem 1 is novel and it has not been reported in the literature. It is important to note that the new findings mathematically consist of the modified Bessel function of the first kind under the denominator, which implies that exact mathematical simplification can become difficult for the corresponding cumulative distribution function (cdf) of Rayleigh fading under unified-oCSI conditions. As such, over the large-Σ regime, an asymptotic analysis may be performed.

Lemma 1: The findings in this paper can be simplified for $s_n^2 = 0, n \geq 3$, i.e. Nakagami-m fading under oCSI conditions using $I_{v-\frac{1}{2}}\left(\frac{z}{2}\right) = \frac{\left(\frac{z}{4}\right)^{v-\frac{1}{2}} e^{-\frac{z}{2}} {}_1F_1(v;2v;z)}{\Gamma(v+\frac{1}{2})}$. It is noted that the findings in this paper are valid for $n \geq 3$, or $m \geq 1.5$ because of the function $\Gamma(m-1)$ under the constant Δ. As such, the proposed findings do **not** mathematically reduce to the joint pdf of bivariate exp.c. Rician fading for $n = 2$, despite their similar mathematical form.

Lemma 2: The infinite summations under Theorem 1 do converge for a finite number of terms T.

Proof: The convergence error can be given by

$$E_r = \sum_{k,k_1=T}^{\infty} \frac{\mathcal{S}\gamma^{\frac{\varpi-m+1}{2}} I_\varpi\left(2\delta_2\sqrt{\gamma}\right) e^{-\varrho\epsilon\Gamma_a\gamma} {}_1F_1\left(\mathcal{A}; A\gamma\right)}{I_{m-1}\left(2\delta_4\sqrt{\gamma}\right)}. \tag{6}$$

Because of the followings (i) $e^{-\epsilon\Gamma_a\varrho} \to 0, \varrho \to \infty$, (ii) $\Sigma_2 \to \infty$ for $0 \leq \epsilon < 1$, (iii) $\delta_2, \delta_3, \delta_4 \to 0$, $f(\gamma)$ thus does converge for a finite number of terms T. Moreover, the $s_n^2 > 0$ condition also helps fasten the convergence process. It is noted that convergence of infinite summations under correlated fading has been well-known as shown in [12]. Additionally, $\Delta \to 0$ for $s_n \to \infty$, which ensures the convergence of $f(\gamma)$ and completes the proof. ∎

IV. Discussion

A. Validation

From Fig. 2, it can be observed that (i) perfect matching for pdfs of Rayleigh fading under unified-oCSI, MRC and SC deployment can be evidently seen, which validates the proposed findings and (ii) over high-γ regime, MRC tends to perform better than SC.

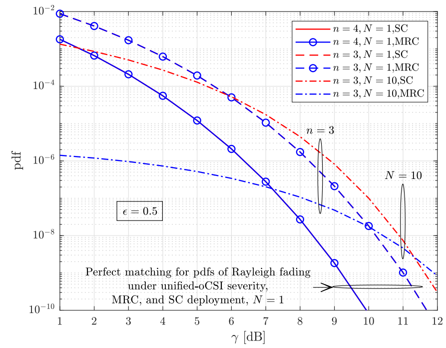

Fig. 2. Cross-verification with perfect matching for pdfs of Rayleigh fading under unified-oCSI severity, MRC and SC deployment versus γ, $N = 1, 10$, $\Upsilon = \Upsilon_t = \Sigma = 1$

B. ECC

The ECC—which can be computed using $W = \int_0^\infty f(\gamma)\log_2(1+\gamma)d\gamma$, where $f(\gamma)$ is given under Theorem 1—under unified-oCSI conditions employing SC has been chosen to be the performance indicator for this paper. The ECC is analysed by varying several parameters, from which additional insight can be gained.

Remark 2: Because of the presence of double nested infinite summations and $I_\varpi(x)$ in the denominator, ECC computation appears to be approximately performed. ECC optimisation has appeared intractable.

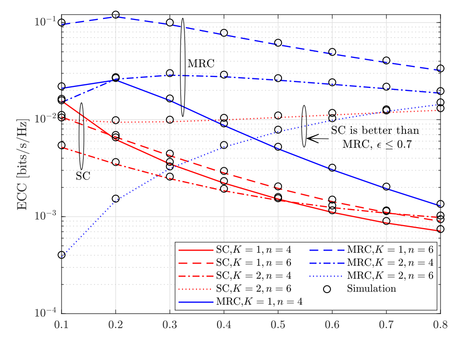

Fig. 3. ECC versus ϵ, $\Upsilon = \Upsilon_t = \Sigma = 1$

1) Versus ϵ: From Fig. 3, it can be seen that (i) increasing system coherence reduces the ECC under specific conditions. Under MRC deployment, the ECC appears to saturate toward a finite value which is much higher than that obtained under SC deployment, which thus shows the benefit of MRC deployment. Under SC deployment, the ECC sharply decreases as ϵ is increased for $K \leq 1, n = 4, 6$ and (ii) for $K \leq 2, n \geq 6$, under both MRC and SC deployment, the ECC steep rises. In addition, SC appears to outperform MRC for $\epsilon \leq 0.7$. For $\epsilon > 0.7$, MRC appears to give higher ECC than SC because relay coherence is sufficient. It is emphasised that an MRC receiver relies on sufficient system coherence to sum individual branch fading gains, which is much more complicated than SC, which relies on the strongest branch for transmission. As such, MRC appears to be sensitive to system coherence as can be clearly seen from Fig. 3.

2) Versus s_n: From Fig. 4, it can be seen that (i) under MRC deployment, keep increasing s_n reduces the

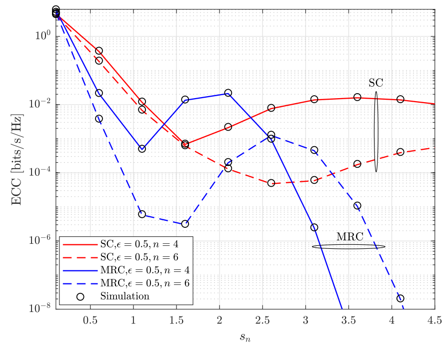

Fig. 4. ECC versus s_n, $\Upsilon = \Upsilon_t = \Sigma = 1$

fading severity but also lowers the ECC. Whereas, under SC deployment, even though for $1.5 \leq s_n \leq 3.5$ MRC outperforms SC, keep increasing s_n saturates the ECC, which is larger than that obtained under MRC deployment. This is because MRC can better-combat fading than SC, resulting in this phenomenon, (ii) it has appeared that increasing the DoF reduces the ECC, which is also a well-known fact, because fading severity is inversely proportional to n, (iii) it clearly appears that the ECC under MRC deployment attains its minimum and maximum over the range of $1.5 \leq s_n \leq 3.5$. Over the large-s_n regime, i.e. strong LoS power, the ECC steeply decreases. Similarly, over the $s_n \leq 1.1$ regime, under MRC deployment, the ECC reaches its minimum, however, as s_n is increased, the ECC attains its maximum over the $2 \leq s_n \leq 2.5$ regime and (iv) increasing s_n even though appears desirable for transmission effectiveness, reduces the fading severity, which lessens the effectiveness of MRC, hence lowering the ECC.

V. Conclusion

The pdf of Rayleigh fading for MRC deployment has been derived in this paper under unified-oCSI conditions considering both relay outdatedness LoS power and DoF for $n \geq 3$ DoF. Unavoidable double-nested infinite summations have been employed, which have been shown to converge for a finite number of terms as shown in this paper. Thorough comparisons for ECC obtained under MRC and SC deployment have been made, which shows the benefit of MRC over SC under most scenarios. The ECC has also been shown to be inversely proportional to ϵ, s_n. Future work on a pdf of relay fading under relay outdatedness and relay imperfection will be reported in a separate publication.

Appendix

Employing MRC, the pdf of DF OR i.i.d. Rayleigh fading operating environment has $2N$-DoF chi-square distribution, which is given by [14, Eq. (2.3–21)] $f_{\gamma_t}(\gamma_t) = \frac{\gamma_t^{N-1} e^{-\Sigma_1 \gamma_t}}{\Gamma(N)\Sigma^N}$. The pdf of Rayleigh fading operating environment under unified-oCSI conditions can be given by [12, Eq. (18)]

$$\begin{aligned}
f(\gamma) &= \int_0^\infty f(\gamma_t|\gamma) f_{\gamma_t}(\gamma_t) d\gamma_t \\
&= \mathcal{Y} \sum_{k=0}^{\infty} \int_0^\infty I_\varpi\left(2\delta_1\sqrt{\gamma\gamma_t}\right) I_\varpi\left(2\delta_2\sqrt{\gamma}\right) I_\varpi\left(2\delta_3\sqrt{\gamma_t}\right) \\
&\quad \times \frac{e^{-\varrho(\epsilon\Gamma_a\gamma+\Gamma_t\gamma_t)}}{\gamma^{\frac{m-1}{2}} I_{m-1}\left(2\delta_4\sqrt{\gamma}\right)} \left(\gamma_t^{N-1} e^{-\Sigma_1\gamma_t}\right) d\gamma_t \\
&= \mathcal{Y} \sum_{k=0}^{\infty} I_\varpi\left(2\delta_2\sqrt{\gamma}\right) \frac{e^{-\varrho\epsilon\Gamma_a\gamma}}{\gamma^{\frac{m-1}{2}} I_{m-1}\left(2\delta_4\sqrt{\gamma}\right)} \\
&\quad \times \int_0^\infty \gamma_t^{N-1} I_\varpi\left(2\delta_1\sqrt{\gamma\gamma_t}\right) I_\varpi\left(2\delta_3\sqrt{\gamma_t}\right) e^{-\Sigma_2\gamma_t} d\gamma_t \\
&= \mathcal{Y} \sum_{k,k_1=0}^{\infty} I_\varpi\left(2\delta_2\sqrt{\gamma}\right) \frac{e^{-\varrho\epsilon\Gamma_a\gamma}\delta_3^{\varpi+2k_1}}{\gamma^{\frac{m-1}{2}} I_{m-1}\left(2\delta_4\sqrt{\gamma}\right)} \\
&\quad \times \int_0^\infty \gamma_t^{N-1} \frac{I_\varpi\left(2\delta_1\sqrt{\gamma\gamma_t}\right)\gamma_t^{\frac{\varpi}{2}+k_1}}{k_1!\Gamma(k_1+\varpi+1)} e^{-\Sigma_2\gamma_t} d\gamma_t,
\end{aligned} \tag{7}$$

which completes the proof using [15, Eq. (3.15.2.5), p. 318, Vol. 4].

References

[1] M. R. Zamani, M. Eslami, M. Khorramizadeh, and Z. Ding, “Energy-efficient power allocation for NOMA with imperfect CSI”, *IEEE Trans. Veh. Technol.*, vol. 68, no. 1, pp. 1009–1013, Jan. 2019.

[2] Y. Zhang, J. Zhang, M. Di Renzo, H. Xiao, and B. Ai, “Reconfigurable intelligent surfaces with outdated channel state information: Centralized vs. distributed deployments”, *IEEE Trans. Commun.*, vol. 70, no. 4, pp. 2742–2756, Apr. 2022.

[3] A. Thakur, A. Kumar, N. Gupta, and P. Chatterjee, “Secrecy analysis of reconfigurable underlay cognitive radio networks with swipt and imperfect csi”, *IEEE Trans. Netw. Sci. Eng.*, vol. 9, no. 1, pp. 89–97, Jan. 2022.

[4] S. Mashhadi, N. Ghiasi, S. Farahmand, and S. M. Razavizadeh, “Deep reinforcement learning based adaptive modulation with outdated csi”, *IEEE Commun. Lett.*, vol. 25, no. 10, pp. 3291–3295, Oct. 2021.

[5] L. Liu, L. Cai, L. Ma, and G. Qiao, “Channel state information prediction for adaptive underwater acoustic downlink ofdma system: Deep neural networks based approach”, *IEEE Trans. Veh. Technol.*, vol. 70, no. 9, pp. 9063–9076, Sept. 2021.

[6] M. Torabi and D. Haccoun, “Capacity analysis of opportunistic relaying in cooperative systems with outdated channel information”, *IEEE Commun. Lett.*, vol. 14, no. 12, pp. 1137–1139, Dec. 2010.

[7] C.-H. Yu and O. Tirkkonen, “Opportunistic multiple relay selection with diverse mean channel gains”, *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 885–891, Mar. 2012.

[8] S. Yadav and P. K. Upadhyay, “Impact of Outdated Channel Estimates on Opportunistic Two-Way ANC-Based Relaying with Three-Phase Transmissions”, *IEEE Trans. Veh. Technol.*, vol. 64, no. 12, pp. 5750–5766, Dec. 2015.

[9] J. Vicario, A. Bel, J. A. Lopez-salcedo, and G. Seco, “Opportunistic relay selection with outdated CSI: outage probability and diversity analysis”, *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2872–2876, June 2009.

[10] D. S. Michalopoulos, N. D. Chatzidiamantis, R. S. Schober, and G. K. Karagiannidis, “Relay selection with outdated channel estimates in Nakagami-m fading”, *in IEEE Int. Conf. Commun.*, June 2011.

[11] L. Sun Y. Li, Q. Yin, H. Chen, and H.-M. Wang, “A channel quality metric in opportunistic selection with outdated CSI over Nakagami-m fading channels”, *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1427–1432, Mar. 2012.

[12] K. S. Miller, *Multidimensional Gaussian Distributions*, Wiley, New York, 1964.

[13] M. S. Alouini and A. Goldsmith, "Adaptive modulation over Nakagami fading channels", *Wireless Personal Commun.*, vol. 13, no. 11, pp. 119–143, Jan. 2000.

[14] J. G. Proakis, *Digital Communications*, McGraw Hill, New York, 2001.

[15] A. P. Prudnikov, Y. A. Brychov, and O. I. Marichev, *Integrals and Series — Volume 1: Elementary Functions*, Gordon and Breach Science, New York, 1986.