ZCZ Orthogonal Finite-Length Sequences for Underground Coded Sonar

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***Abstract*—Zero-correlation-zone (ZCZ) orthogonal finite-length sequence pairs of complex values or real values are proposed for underground multiplexing coded sonar. Primary orthogonal sequence pair constructed from element sequences is lengthened by zero-value insertion and convolved with a real-valued common sequence and leads to orthogonal sequence pair. Polyphase sequences or low magnitude real-valued sequences are used for transmitting at efficient drive power. Process gain is decided by the zero-value insertion length. The sequence pairs can be transmitted by single or no carrier.**

Keywords—Huffman sequence, ZCZ, orthogonal, finite-length, polyphase, complex value, real value, process gain, underground multiplexing coded sonar.

I. Introduction

A chirp coded elastic wave is adopted for intensifying the energy transmission through lossy path in underground prove, where for computer tomography, transmission characteristics through different paths are obtained by using time-division multiplexing of the chirp code for long time interval [1]. For the efficient measuring, parallelly operated wide ZCZ multiplexing system is to be developed. As the similar system, multiple-input multiple-output (MIMO) systems are developed for the applications to mobile phones and automobiles [2], where almost system transmits periodic waveforms that accompanies extra calculations over a period. If the multiplexing system to transmit isolated waveforms is developed, calculations may be saved. Isolated waveforms may be constructed from complementary sequence pairs or Huffman sequence. Mutually orthogonal complementary sequence pairs have been investigated [3], where their application to practical system requires independent transmission of pair sequences and the increase of multiplexing signals reduces to complicate the system. Huffman sequence is a non-two-valued finite-length sequence with impulsive aperiodic autocorrelation function [4]. One of the authors proposed orthogonal Huffman sequences for code-division multiple-access (CDMA), which have narrow ZCZ orthogonality and not-small magnitude of sequence value [5]. We cannot construct wide ZCZ orthogonal and small-magnitude sequences with the same impulsive autocorrelation function. The authors also have developed underground sonar using a Huffman sequence with length 11822 and maximum magnitude 5.2 [6],[7] and have considered ZCZ orthogonal sequences proper to multiplexing one [8].

In this paper, we propose the wide ZCZ orthogonal and small-magnitude sequences by combining complex-valued or real-valued orthogonal Huffman sequence pairs and a real-valued Huffman sequence. The multiplexing system using the sequences can achieve single or no carrier isolated waveform multiple transmission and simultaneous parallel outputs among multiple transmitters and multiple receivers and can be applicable to multiplexing coded radar, sonar, etc. Section II introduce vision of underground multiplexing sonar. Section III develops generalized Huffman sequence. Section VI provides complex-valued or real-valued orthogonal sequence pairs and low-magnitude real-valued sequence. Section V constructs ZCZ orthogonal sequence pairs. Section VI explains signal transmission and processing. Section VII is conclusion.

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II. Vision of Multiplexing Coded Sonar

Fig.1 shows the vision of a multiplexing coded sonar. Fig.1(a) shows positions of transmitters T_0, T_1 and receivers R_0, R_1 and paths of signals from T_0, T_1 to R_0, R_1 ,as underground elevation view. Fig.1(b) shows transmitting signals on T_0, T_1 and detected signals on R_0, R_1 through the paths, where the other signals do not interfere with the detected signals in a specified interval: S_{00} is R_0 output from T_0 to R_0, and S_{01} is R_0 output from T_1 to R_0, and S_{10} is R_1 output from T_0 to R_1, and S_{11} is R_1 output from T_1 to R_1. Transmitters and receivers simultaneously operate and parallelly execute signal processing.

III. Generalized Huffman Sequences

A. Fundamental relations of sequences

From a pair of finite-length transmitting sequence $\{a_{\ell,i}\}$ and reference sequence $\{b_{\ell,i}\}$ of length $M = N + 1$ ($i = 0,1,\cdots,M-1, \ell = 0,1,,\cdots,L-1$) , transmitting and reference codes of time function are derived as

$$a_\ell(\mathrm{t}) = \sum_{i=0}^{N} a_{\ell,i}\delta(t - i\Delta t) \quad (1)$$

$$b_\ell(\mathrm{t}) = \sum_{i=0}^{N} b_{\ell,i}\delta(t - i\Delta t) \quad (2)$$

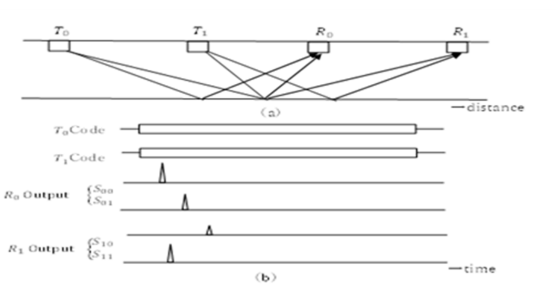

Fig.1. Vision of multiplexing coded sonar.

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where $\delta(t)$ is Dirac's delta function of time t and Δt is time interval between weighted impulse train. The Fourier transforms of (1) and (2) are given by

$$A_\ell = \int_{-\infty}^{+\infty} a_\ell(t) e^{-j2\pi f t}\, dt = \sum_{i=0}^{N} a_{\ell,i} Z^{-i}, \quad (3)$$

$$B_\ell = \int_{-\infty}^{+\infty} b_\ell(t) e^{-j2\pi f t}\, dt = \sum_{i=0}^{N} b_{\ell,i} Z^{-i}, \quad (4)$$

where f is frequency, $Z = e^{j2\pi f \Delta t}$ and $j = \sqrt{-1}$. Cross-spectrum between A_ℓ and B_ℓ is expressed as

$$A_\ell \cdot B_\ell^* = M \sum_{i'=-N}^{N} \rho_{\ell,\ell,i'} Z^{-i'}, \quad (5)$$

where B_ℓ^* is the complex conjugate of B_ℓ, as

$$B_\ell^* = \sum_{i=0}^{N} b_{\ell,i}^* Z^{i} = Z^N \sum_{i=0}^{N} b_{\ell,N-i}^* Z^{-i} \quad (6)$$

and the auto-equivalent correlation function between $\{a_{\ell,i}\}$ and $\{b_{\ell,i}\}$ is expressed as

$$\rho_{\ell,\ell,i'} = \frac{1}{M} \sum_{i=0}^{N} a_{\ell,i} b_{\ell,i-i'}^* . \quad (7)$$

Here, we force the auto-equivalent correlation function to be

$$\rho_{\ell,\ell,i'} = \delta_{i',0} + \varepsilon_N \delta_{i',N} + \varepsilon_{-N} \delta_{i',-N}, \quad (8)$$

which has unity at shift $i' = 0$, ε_N at positive shift end $i' = N$ and ε_{-N} at negative shift end $i' = -N$, where $\delta_{i',0}$, $\delta_{i',N}$ and $\delta_{i',-N}$ denote Kroneckar's deltas. The complex correlation values at positive and negative shift-ends are expressed as

$$\varepsilon_N = \frac{1}{M} a_{\ell,N} b_{\ell,0}^* = |\varepsilon_N| e^{j\varphi_N}, \quad (9)$$

$$\varepsilon_{-N} = \frac{1}{M} a_{\ell,0} b_{\ell,N}^* = |\varepsilon_{-N}| e^{j\varphi_{-N}}, \quad (10)$$

respectively, where φ_N and φ_{-N} are phases of ε_N and ε_{-N}. Thus, the cross-spectrum (5) is decomposed as

$$\begin{aligned} A_\ell \cdot B_\ell^* &= M(\varepsilon_{-N} Z^N + 1 + \varepsilon_N Z^{-N}) \\ &= M\varepsilon_N Z^N (Z^{-N} + e^{S_1+S_2})(Z^{-N} + e^{S_1-S_2}), \quad (11) \end{aligned}$$

where

$$e^{S_1} = \sqrt{\varepsilon_{-N}/\varepsilon_N}, \quad (12)$$

$$coshS_2 = 1/(2\sqrt{\varepsilon_N \varepsilon_{-N}}), \quad (13)$$

and $\varepsilon_N = e^{-S_1}/(2coshS_2)$ and $\varepsilon_{-N} = e^{S_1}/(2coshS_2)$ in reverse.

B. Complex-valued sequence

For $|e^{S_1+S_2}| = |e^{S_1-S_2}| = 1$, we obtain

$$e^{S_1} = e^{j(\varphi_{-N} - \varphi_N)/2}, \quad (14)$$

$$coshS_2 = \cos N\beta = e^{-j(\varphi_N + \varphi_{-N})/2}/(2|\varepsilon_N|), \quad (15)$$

$$|\varepsilon_N| = |\varepsilon_{-N}|, \quad (16)$$

where $S_2 = jN\beta$. Here, we select the same phase relation $\varphi_N = -\varphi_{-N}$ as that of the real valued sequence of $\{a_{\lambda,i}\} = \{b_{\lambda,i}\}$ later on, and we obtain

$$\cos N\beta = 1/(2|\varepsilon_N|), \quad (17)$$

$$1/2 \le |\varepsilon_N| \le 1, \quad (18)$$

$$N\beta = \beta_0 + 2\pi k;\ k = 0,1,\cdots,N-1, \quad (19)$$

where $-\pi/3 \le \beta_0 \le \pi/3$.

In this case, the cross-spectrum (11) is decomposed as

$$\begin{aligned} A_\ell \cdot B_\ell^* &= M\varepsilon_N Z^N \prod_{m=0}^{N-1} (Z^{-1} - \Omega e^{j\beta + j\frac{2m\pi}{N}}) \\ &\times \prod_{m=0}^{N-1} (Z^{-1} - \Omega e^{-j\beta + j\frac{2m\pi}{N}}), \quad (20) \end{aligned}$$

where $\Omega = e^{j\frac{\pi - \varphi_N}{N}}$ and $e^{jN\beta} + e^{-jN\beta} = 1/|\varepsilon_N|$. Equation (20) gives a complex-valued sequence pair of $\{a_{\ell,i}\}$ and $\{b_{\ell,i}^*\}$. The case of $\varphi_N \ne -\varphi_{-N}$ is not discussed here.

C. Real-valed sequence

For real-valued $S_2 = N\alpha$ $(\alpha > 0)$, it holds

$$e^{\pm N\alpha} = coshS_2 \pm sinhS_2 = \frac{1 \pm \sqrt{1 - 4|\varepsilon_N|^2}}{2|\varepsilon_N|}. \quad (21)$$

The cross-spectrum (11) is decomposed as

$$\begin{aligned} A_\ell \cdot B_\ell^* &= M\varepsilon_N Z^N \prod_{m=0}^{N-1} (Z^{-1} - \Omega e^{\alpha + j\frac{2m\pi}{N}}) \\ &\times \prod_{m=0}^{N-1} (Z^{-1} - \Omega e^{-\alpha + j\frac{2m\pi}{N}}), \quad (22) \end{aligned}$$

where $\Omega = e^{j\frac{\pi - \varphi_N}{N}}$ and $e^{N\alpha} + e^{-N\alpha} = 1/|\varepsilon_N|$[5]. Equation (22) gives a real-valued sequence $\{a_{\ell,i}\} = \{b_{\ell,i}\}$ in principle, but a pair of real-valued sequences of $\{a_{\ell,i}\}$ and $\{b_{\ell,i}\}$ for intended purpose.

IV. Orthogonal Pairs and Related Sequence

A. Orthogonal pairs of complex-valued sequences

For the transmission of signals with low amplitude deviation, polyphase sequence is introduced. For convenience, we construct a pair of sequences for length $M = 9$ $(N = 8)$. From (20), we have the expression

$$\begin{aligned} A_\ell \cdot B_\ell^* &= M\varepsilon_N Z^N (Z^{-1} - w_1)(Z^{-1} + w_1) \\ &\times (Z^{-2} + w_1^2)(Z^{-4} + w_1^4)(Z^{-1} - w_2)(Z^{-1} + w_2) \\ &\times (Z^{-2} + w_2^2)(Z^{-2} + w_2^4) \quad (23) \end{aligned}$$

where $w_1 = w\Omega, w_2 = w^{-1}\Omega$ and $w = e^{j\beta}$, and hence we select four pairs of sequence spectra

$$A_0 = \sqrt{M|\varepsilon_N|} Z^{-1}(Z^{-1} + w_1)(Z^{-2} + w_1^2)(Z^{-4} + w_1^4), (24)$$

$$\begin{aligned} B_o^* &= \sqrt{M|\varepsilon_N|} e^{j\varphi_N} Z^{N+1} (Z^{-1} - w_1)(Z^{-1} - w_2) \\ &\times (Z^{-1} + w_2)(Z^{-2} + w_2^2)(Z^{-4} + w_2^4), \quad (25) \end{aligned}$$

$$A_1 = \sqrt{M|\varepsilon_N|} Z^{-1}(Z^{-1} + w_1)(Z^{-2} + w_2^2)(Z^{-4} + w_1^4), (26)$$

$$\begin{aligned} B_1^* &= \sqrt{M|\varepsilon_N|} e^{j\varphi_N} Z^{N+1} (Z^{-1} - w_1)(Z^{-1} - w_2) \\ &\times (Z^{-1} + w_2)(Z^{-2} + w_1^2)(Z^{-4} + w_2^4), \quad (27) \end{aligned}$$

$$A_2 = \sqrt{M|\varepsilon_N|} Z^{-1}(Z^{-1} + w_1)(Z^{-2} + w_1^2)(Z^{-4} + w_2^4), (28)$$

$$\begin{aligned} B_2^* &= \sqrt{M|\varepsilon_N|} e^{j\varphi_N} Z^{N+1} (Z^{-1} - w_1)(Z^{-1} - w_2) \\ &\times (Z^{-1} + w_2)(Z^{-2} + w_2^2)(Z^{-4} + w_1^4), \quad (29) \end{aligned}$$

$$A_3 = \sqrt{M|\varepsilon_N|} Z^{-1}(Z^{-1} + w_1)(Z^{-2} + w_2^2)(Z^{-4} + w_2^4), (30)$$

$$\begin{aligned} B_3^* &= \sqrt{M|\varepsilon_N|} e^{j\varphi_N} Z^{N+1} (Z^{-1} - w_1)(Z^{-1} - w_2) \\ &\times (Z^{-1} + w_2)(Z^{-2} + w_1^2)(Z^{-4} + w_1^4), \quad (31) \end{aligned}$$

where all spectra of parameters alternating between w_1 and w_2 correspond to those of sequences inverted in index. Equations (24) to (27) are expanded by polynomials as

$$A_0 = \sqrt{M|\varepsilon_N|}Z^{-1}(Z^{-7} + w_1 Z^{-6} + w_1^2 Z^{-5} + w_1^3 Z^{-4} + w_1^4 Z^{-3} + w_1^5 Z^{-2} + w_1^6 Z^{-1} + w_1^7) \quad (32)$$

$$B_o^* = \sqrt{M|\varepsilon_N|}e^{j\varphi_N}Z^{N+1}(Z^{-9} - w_1 Z^{-8} - w_2^8 Z^{-1} + w_1 w_2^8) \quad (33)$$

$$A_1 = \sqrt{M|\varepsilon_N|}Z^{-1}(Z^{-7} + w_1 Z^{-6} + w_2^2 Z^{-5} + w_1 w_2^2 Z^{-4} + w_1^4 Z^{-3} + w_1^5 Z^{-2} + w_1^4 w_2^2 Z^{-1} + w_1^5 w_2^2) \quad (34)$$

$$B_1^* = \sqrt{M|\varepsilon_N|}e^{j\varphi_N}Z^{N+1}\{Z^{-9} - w_1 Z^{-8} + (w_1^2 - w_2^2)Z^{-7} - w_1(w_1^2 - w_2^2)Z^{-6} - w_2^2(w_1^2 - w_2^2)Z^{-5} + w_1 w_2^2(w_1^2 - w_2^2)Z^{-4} + w_2^4(w_1^2 - w_2^2)Z^{-3} - w_1 w_2^4(w_1^2 - w_2^2)Z^{-2} - w_1^2 w_2^6 Z^{-1} + w_1^3 w_2^6\}, \quad (35)$$

and those lead sequence pairs as

$$\{a_{0,i}\} = \sqrt{M|\varepsilon_N|}[0, w_1^7, w_1^6, w_1^5, w_1^4, w_1^3, w_1^2, w_1, 1, 0], \quad (36)$$

$$\{b_{0,i}^*\} = \sqrt{M|\varepsilon_N|}e^{j\varphi_N}[1, -w_1, 0,0,0,0,0,0, -w_2^8, w_1 w_2^8], \quad (37)$$

$$\{a_{1,i}\} = \sqrt{M|\varepsilon_N|}[0, w_1^5 w_2^2, w_1^4 w_2^2, w_1^5, w_1^4, w_1 w_2^2, w_2^2, w_1, 1, 0], \quad (38)$$

$$\{b_{1,i}^*\} = \sqrt{M|\varepsilon_N|}e^{j\varphi_N}[1, -w_1, (w_1^2 - w_2^2), -w_1(w_1^2 - w_2^2), -w_2^2(w_1^2 - w_2^2), w_1 w_2^2(w_1^2 - w_2^2), w_2^4(w_1^2 - w_2^2), -w_1 w_2^4(w_1^2 - w_2^2), -w_1^2 w_2^6, w_1^3 w_2^6] \quad (39)$$

where $\{a_{0,i}\}$ and $\{a_{1,i}\}$ start at one chip delay, and $\{b_{0,i}^*\}$ and $\{b_{1,i}^*\}$ conform to such relation as (6).

Cross-spectra between sequence pairs are given by

$$A_0 \cdot B_1^* = A_2 \cdot B_3^* = M\varepsilon_N Z^N (Z^{-2} + w_1^2)(Z^{-2} - w_2^2) \times (Z^{-4} + w_2^4)(Z^{-8} - w_1^8), \quad (40)$$

$$A_0 \cdot B_2^* = A_1 \cdot B_3^* = M\varepsilon_N Z^N (Z^{-4} + w_1^4)(Z^{-4} - w_2^4)(Z^{-8} - w_1^8), \quad (41)$$

$$A_0 \cdot B_3^* = M\varepsilon_N Z^N (Z^{-2} + w_1^2)(Z^{-2} - w_2^2) \times (Z^{-4} + w_1^4)(Z^{-8} - w_1^8), \quad (42)$$

$$A_1 \cdot B_2^* = M\varepsilon_N Z^N (Z^{-2} - w_1^2)(Z^{-2} + w_2^2) \times (Z^{-4} - w_2^4)(Z^{-4} + w_1^4)^2, \quad (43)$$

where the factors of $A_{\ell_1} \cdot B_{\ell_2}^*$ and $A_{\ell_2} \cdot B_{\ell_1}^*$ have w_1 and w_2 exchanged. Let a set of two pairs of $\{a_{\ell_1,i}\}$, $\{b_{\ell_1,i}^*\}$ and $\{a_{\ell_2,i}\}$, $\{b_{\ell_2,i}^*\}$ represent (ℓ_1, ℓ_2) and then pair sets (0,1), (1,2), (2,3), (0,3) are orthogonal at shift $i' = 1\ (mod 2)$, and the other sets (0,2), (1,3) are orthogonal at shift $i' = 1, 2, 3\ (mod 4)$. Thus, we obtain a set of orthogonal four sequence pairs $< \{a_{0,i}\}, \{b_{0,i}^*\} >$, $< \{a_{1,i-1}\}, \{b_{1,i-1}^*\} >$, $< \{a_{2,i-2}\}, \{b_{2,i-2}^*\} >$, $< \{a_{3,,i-3}\}, \{b_{3,i-3}^*\} >$. In the above, we can obtain another set of orthogonal pairs by replacing w_1 by w_2. Generally, we have a set of orthogonal sequence pairs $< \{a_{\ell,i-\ell}\}, \{b_{\ell,i-\ell}^*\} > \left(\ell = 0, 1, \cdots, \frac{N}{2} - 1\right)$ for $N = 2^\nu; \nu = 2, 3, 4, \cdots$ [8]. In order to understand correlation function, we call the correlation function between the desired transmitting sequence $\{a_{\ell',i-\ell'}\}$ and the desired reference sequence $\{b_{\ell',i-\ell'}^*\}$ the auto-equivalent correlation function and that between the undesired transmitting sequence $\{a_{\ell,i-\ell}\}$ and the desired reference sequence $\{b_{\ell',i-\ell'}^*\}$ the cross-equivalent correlation function. Let us show the concrete example. Eq. (40) is expanded as

$$A_0 \cdot B_1^* = M\varepsilon_N Z^8\{Z^{-16} + (w_1^2 - w_2^2)Z^{-14} - w_2^2(w_1^2 - w_2^2)Z^{-12} + w_2^4(w_1^2 - w_2^2)Z^{-10} - w_1^2(w_1^6 + w_2^6)Z^{-8} - w_1^8(w_1^2 - w_2^2)Z^{-6} + w_1^8 w_2^2(w_1^2 - w_2^2)Z^{-4} - w_1^8 w_2^4(w_1^2 - w_2^2)Z^{-2} + w_1^{10} w_2^6\} \quad (44)$$

and the cross-spectrum between $\{a_{0,i}\}$ and $\{b_{1,i-1}^*\}$ becomes $A_0 \cdot B_1^* Z$. Then we obtain the cross-equivalent correlation function between $\{a_{0,i}\}$ and $\{b_{1,i-1}^*\}$ as

$$\rho_{0,1,i'} = \frac{1}{M}\sum_{i=0}^{N+1} a_{0,i} b_{1,i-i'-1}^* = \varepsilon_N\{w_1^{10} w_2^6 \delta_{i',-9} - w_1^8 w_2^4(w_1^2 - w_2^2)\delta_{i',-7} + w_1^8 w_2^2(w_1^2 - w_2^2)\delta_{i',-5} - w_1^8(w_1^2 - w_2^2)\delta_{i',-3} - w_1^2(w_1^6 + w_2^6)\delta_{i',-1} + w_2^4(w_1^2 - w_2^2)\delta_{i',1} - w_2^2(w_1^2 - w_2^2)\delta_{i',3} + (w_1^2 - w_2^2)\delta_{i',5} + \delta_{i',7}\} \quad (45)$$

and expresses orthogonality at shift $i' = 0$, where for $N = 8$ and $\beta_0 = \pi/8$, it follows $|w_1^2 - w_2^2| = 2sin\left(\frac{2\beta_0}{N}\right) \cong 0.196$, $|w_1^6 + w_2^6| = 2cos\left(\frac{6\beta_0}{N}\right) \cong 1.914$, $|\varepsilon_N| = \frac{1}{2cos\beta_0} \cong 0.541$ the maximum absolute value of this cross-equivalent correlation function takes about 1.036 at $i' = -1$. Similarly, the other cross-equivalent correlation functions take 0 at shift $i' = 0$. The peak of cross-equivalent correlation function appears at $i' = -1$ for $\{\rho_{0,1,i'}\}$, $\{\rho_{1,2,i'}\}$ and $\{\rho_{2,3,i'}\}$, at $i' = -2$ for $\{\rho_{0,2,i'}\}$ and $\{\rho_{1,3,i'}\}$ and at $i' = -3$ for $\{\rho_{0,3,i'}\}$. The non-zero values of cross-equivalent correlation functions are eluded for the later synthesized sequences. The auto-equivalent correlation function of shifted sequence pair follows (8). The polyphase sequences $\{a_{\ell,i-\ell}\}$ can be used for making transmitting signals with low deviated amplitude, as mentioned later.

B. Orthogonal pairs of real-valued sequences

In the real-valued sequence with the absolute shift-end correlation value $|\varepsilon_N| = 1/(2coshN\alpha) \equiv e^{-\tilde{\alpha}}/2$; $\tilde{\alpha} \ll 1$, the following holds

$$\tilde{\alpha} \cong (N\alpha)^2/2 \quad (46)$$

for $N\alpha < 0.2$. In this case, similarly as mentioned above, we have real-valued orthogonal sequence pairs for $\Omega = 1(\varphi_N = \pi)$, by replacing $w = e^{j\beta}$ in the above section with $w = e^{\alpha}$. For example, two pairs corresponding to (36) to (39) are replaced with

$$\{a_{0,i}\} = \sqrt{M|\varepsilon_N|}[0, e^{7\alpha}, e^{6\alpha}, e^{5\alpha}, e^{4\alpha}, e^{3\alpha}, e^{2\alpha}, e^{\alpha}, 1, 0], \quad (47)$$

$$\{b_{0,i}\} = -\sqrt{M|\varepsilon_N|}[1, -e^{\alpha}, 0,0,0,0,0,0, -e^{-8\alpha}, e^{-7\alpha}], \quad (48)$$

$$\{a_{1,i}\} = \sqrt{M|\varepsilon_N|}[0, e^{3\alpha}, e^{2\alpha}, e^{5\alpha}, e^{4\alpha}, e^{-\alpha}, e^{-2\alpha}, e^{\alpha}, 1, 0], \quad (49)$$

$$\{b_{1,i}\} = -\sqrt{M|\varepsilon_N|}[1, -e^{\alpha}, (e^{2\alpha} - e^{-2\alpha}),$$

$$-e^{\alpha}(e^{2\alpha}-e^{-2\alpha}),-e^{2\alpha}(e^{2\alpha}-e^{-2\alpha}),e^{-\alpha}(e^{2\alpha}-e^{-2\alpha}),$$
$$e^{-4\alpha}(e^{2\alpha}-e^{-2\alpha}),-e^{-3\alpha}(e^{2\alpha}-e^{-2\alpha}),-e^{-4\alpha},e^{-3\alpha}], \quad (50)$$

where $e^{7\alpha}\cong 1.091, e^{6\alpha}\cong 1.078, e^{5\alpha}\cong 1.065, e^{4\alpha}\cong 1.051,$ $e^{3\alpha}\cong 1.038, e^{2\alpha}\cong 1.025, e^{\alpha}\cong 1.013, e^{-8\alpha}\cong 0.905, e^{-7\alpha}\cong$ $0.916, |\varepsilon_N|\cong 0.498$ for $N\alpha=0.1, \varphi_N=\pi$. Transmission of these sequences does not always need sine and cosine waves as for modulating complex values .

C. Real-valued sequence of long-length

For real-valued sequence with shift-end correlation absolute value $|\varepsilon_N|\ll 1/2$, four formulae are given according to positive or negative ε_N and even or odd length M [5]. The sequence spectrum for $\varepsilon_N<0$ and odd length is given by

$$A_\lambda=B_\lambda=-\sqrt{M|\varepsilon_N|}K_\lambda(Z^{-1}-w_0)(Z^{-1}-w_{N/2})$$
$$\times\prod_{m=1}^{N/2-1}(Z^{-2}-2w_mZ^{-1}cos\frac{2m\pi}{N}+w_m^2) \quad (51)$$

where $w_m=e^{\alpha}\ or\ e^{-\alpha}$ and $K_\lambda=1/\sqrt{w_0w_{N/2}\prod_{m=1}^{N/2-1}w_m}$.

The paper[5] presents ZCZ orthogonal sequence set from (51), where ZCZ, sequence magnitude and family number are restricted by tradeoff relations. The paper[6] presents a sequence of positive ε_N and even long length M due to parameter selection derived from quadratic residue, where the sequence magnitude gradually increases with the length and reaches 5.2 with $M=11822$.

Here, we aim to obtain single real-valued sequence with long length and low sequence magnitude. From parameter combinations of (51) for $N=M-1=2p$, prime $p=1(mod4)$, a spectrum of a real valued sequence is derived

$$A_\lambda=-\sqrt{M|\varepsilon_N|}K_\lambda(Z^{-2}-sZ^{-1}-1)$$
$$\times\prod_{m=1}^{(p-1)/2}\{Z^{-4}-2sx_mZ^{-3}cos\frac{2m\pi}{p}$$
$$+(s^2-2cos\frac{4m\pi}{p})Z^{-2}+2sx_mZ^{-1}cos\frac{2m\pi}{p}+1\}, \quad (52)$$

where $s=e^{\alpha}-e^{-\alpha}$ and $\{x_m\}(m=1,2,\cdots,p-1)$ is quadratic residue sequence from prime $p=1(mod4)$[6],as shown in Table I ($+$ and $-$ denote $+1$and -1, respectively). An example of the sequence is given by

$$\{a_{\lambda,i}\}=\{b_{\lambda,i}\}=[0.724,\ 1.447,\ 1.447, 0.341, -0.764,$$
$$-0.422,\ 0.764, 0.341, -1.447, 1.447, -0.724]$$

TABLE I. QUADRATIC RESIDUE SEQUENCE

p	$\{x_m; m=1,2,,\cdots,p-1\}$
5	$+--+$
13	$+-++----++-+$
17	$++-+---++----+-++$

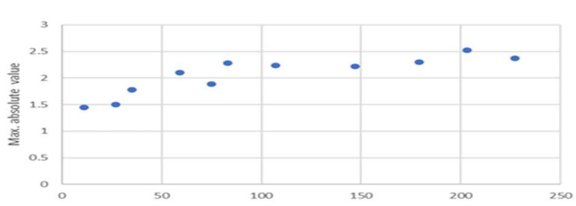

Fig.2. Maximum absolute value of sequence.

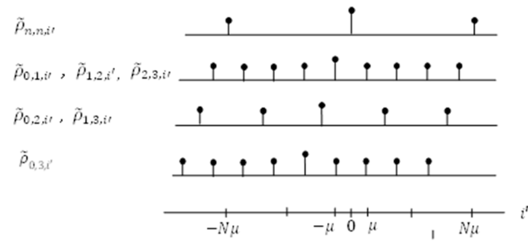

Fig.3 Correlation image between zero-value inserted sequences.

where $N=10$, $|\varepsilon_N|=1/21, \varphi_N=\pi$ and $\alpha\cong 0.3042$. Fig. 2 shows the maximum absolute value versus sequence length, calculated from (52) adjusting the value of $|\varepsilon_N|$, where the sequence magnitude is below about 2.5. In the next section the real-valued sequence is included in sequence construction and reflects the magnitude of transmitting sequence. As an example, this sequence is introduced, though the smaller magnitude real-valued sequence may be developed in the future.

V. SYNTHESIS OF ORTHOGONAL ZCZ SEQUENCES

A. Correlation functions of zero-value inserted sequences

We insert zero values to the orthogonal sequence pair $<\{a_{\ell,i-\ell}\},\{b^*_{\ell,i-\ell}\}>\left(\ell=0,1,\cdots,\frac{N}{2}-1\right)$ as

$$\tilde{a}_{\ell,i}=\sum_{i_1=1+\ell}^{N+\ell}a_{\ell,i_1-\ell}\,\delta_{i,\mu i_1} \quad (53)$$
$$\tilde{b}^*_{\ell,i}=\sum_{i_2=\ell}^{N+1+\ell}b^*_{\ell,i_2-\ell}\,\delta_{i,\mu i_2}\ , \quad (54)$$

where $\mu=1,2,3,\cdots$. The auto-equivalent correlation function of $\{\tilde{a}_{\ell,i}\}$ and $\{\tilde{b}^*_{\ell,i}\}$ is reduced to

$$\tilde{\rho}_{\ell,\ell,i'}=\frac{1}{M}\sum_{i_0=0}^{\mu(N+1+\ell)}\tilde{a}_{\ell,i_0}\,\tilde{b}^*_{\ell,i_0-i'}=\frac{1}{M}(\tilde{a}_{\ell,i'}\odot\tilde{b}^*_{\ell,-i'})$$
$$=\delta_{i',0}+\varepsilon_N\delta_{i',N\mu}+\varepsilon_{-N}\delta_{i',-N\mu}, \quad (55)$$

where $\odot$ denotes convolution. The cross-equivalent correlation function between $\{\tilde{a}_{0,i}\}$ and $\{\tilde{b}^*_{1,i}\}$, for example, corresponding to (45) is given by

$$\tilde{\rho}_{0,1,i'}=\frac{1}{M}(\tilde{a}_{0,i'}\odot\tilde{b}^*_{1,-i'})=\varepsilon_N\{w_1^{10}w_2^6\delta_{i',-9\mu}$$
$$-w_1^8w_2^4(w_1^2-w_2^2)\delta_{i',-7\mu}+w_1^8w_2^2(w_1^2-w_2^2)\delta_{i',-5\mu}$$
$$-w_1^8(w_1^2-w_2^2)\delta_{i',-3\mu}-w_1^2(w_1^6+w_2^6)\delta_{i',-\mu}$$
$$+w_2^4(w_1^2-w_2^2)\delta_{i',\mu}-w_2^2(w_1^2-w_2^2)\delta_{i',3\mu}$$
$$+(w_1^2-w_2^2)\delta_{i',5\mu}+\delta_{i',7\mu}\}\ . \quad (56)$$

Similarly the other cross-equivalent correlation function takes non-zero value at shift $i'=\pm\mu\kappa$ and 0 at shift $i'\neq\pm\mu\kappa$ $(\kappa=1,2,\cdots,N+N/2-1)$,where $\pm$ sign corresponds to combination of sequences such as $\tilde{\rho}_{1,0,i'}=\tilde{\rho}^*_{0,1,-i'}$. Fig.3 shows the image of auto-equivalent and cross-equivalent correlation functions, based on the former correlation functions. Thus sequence pair $<\{\tilde{a}_{\ell,i}\},\{\tilde{b}^*_{\ell,i}\}>$ is orthogonal at $[-(\mu-1),(\mu-1)]$.

B. Correlation functions of convolved sequence

We prepare the real-valued sequence $\{a'_{\lambda,i}\}=\{b'_{\lambda,i}\}$with length $M'=N'+1$ whose autocorrelation function is

$$\rho'_{\lambda,\lambda,i'} = \frac{1}{M'}(a'_{\lambda,i'} \odot a'_{\lambda,-i'})$$
$$= \delta_{i',0} + \varepsilon'_{N'}\delta_{i',N'} + \varepsilon'_{-N'}\delta_{i',-N'} \quad (57)$$

and make the transmitting sequence $\{\hat{a}_{\ell,i}\}$ by convolving it to $\{\tilde{a}_{\ell,i}\}$, and the reference sequence $\{\hat{b}^*_{\ell,i}\}$ by convolving it to $\{\tilde{b}^*_{\ell,i}\}$. The auto-equivalent correlation function of $\{\hat{a}_{\ell,i}\}$ and $\{\hat{b}^*_{\ell,i}\}$ is given by

$$\hat{\rho}_{\ell,\ell,i'} = \frac{K}{\hat{M}}(\tilde{a}_{\ell,i'} \odot a'_{\lambda,i'}) \odot (\tilde{b}^*_{\ell,-i'} \odot a'_{\lambda,-i'})$$
$$= \frac{K}{\hat{M}}(a'_{\lambda,i'} \odot a'_{\lambda,-i'}) \odot (\tilde{a}_{\ell,i'} \odot \tilde{b}^*_{\ell,-i'})$$
$$= (\delta_{i',0} + \varepsilon'_{N'}\delta_{i',N'} + \varepsilon'_{-N'}\delta_{i',-N'})$$
$$\odot(\delta_{i',0} + \varepsilon_N\delta_{i',N\mu} + \varepsilon_{-N}\delta_{i',-N\mu})$$
$$= \delta_{i',0} + \varepsilon'_{N'}\delta_{i',N'} + \varepsilon'_{-N'}\varepsilon_N\delta_{i',N\mu-N'} + \varepsilon_N\delta_{i',N\mu}$$
$$+\varepsilon'_{N'}\varepsilon_N\delta_{i',N\mu+N'} + \varepsilon'_{-N'}\delta_{i',-N'} + \varepsilon'_{N'}\varepsilon_{-N}\delta_{i',-N\mu+N'}$$
$$+\varepsilon_{-N}\delta_{i',-N\mu} + \varepsilon'_{-N'}\varepsilon_{-N}\delta_{i',-N\mu-N'}, \quad (58)$$

where the average length of synthesized sequence is $\hat{M} = N\mu + M'$ and normalizing coefficient to give $\hat{\rho}_{\ell,\ell,i'}$=1 at $i' = 0$ is

$$K = \hat{M}/(M'M)\ . \quad (59)$$

The auto-equivalent correlation function at the peak area

$$\hat{\rho}_{\ell,\ell,i'} = \delta_{i',0} + \varepsilon'_{N'}\delta_{i',N'} + \varepsilon'_{-N'}\delta_{i',-N'} \quad (60)$$

where $-N' \le i' \le N'$. The cross-equivalent correlation function of $\{\hat{a}_{\ell,i}\}$ and $\{\hat{b}^*_{\ell',i}\}$ is given by

$$\hat{\rho}_{\ell,\ell',i'} = \frac{K}{\hat{M}}(a'_{i'} \odot a'_{-i'}) \odot (\tilde{a}_{\ell,i'} \odot \tilde{b}^*_{\ell',-i'})$$
$$= (\delta_{i',0} + \varepsilon'_{N'}\delta_{i',N'} + \varepsilon'_{-N'}\delta_{i',-N'}) \odot \tilde{\rho}_{\ell,\ell',i'} \quad (61)$$

and at the area $-\mu < i' < \mu$

$$\hat{\rho}_{\ell,\ell',i'} = \tilde{\rho}_{\ell,\ell',-\mu}\varepsilon'_{N'}\delta_{i',-\mu+N'} + \tilde{\rho}_{\ell,\ell',\mu}\varepsilon'_{-N'}\delta_{i',\mu-N'}. \quad (62)$$

If we select $N' = 3\mu/2$, we obtain the maximum orthogonal area as

$$[-\mu/2+1,\ \mu/2-1]\ . \quad (63)$$

If $\mu + 2 \le N' \le 3\mu/2$, we obtain the less orthogonal area

$$[\mu - N' + 1,\ -\mu + N' - 1]\ , \quad (64)$$

where N' should be as near to $3\mu/2$ as possible. Fig.2 will help to understand these relations. Correlation processing is effective around the area of (63).

C. Process gain on signal detection

Suppose signals $\{\hat{a}_{\ell,i}\}(\ell = 1,\cdots,\frac{N}{2}-1)$ are transmitted from the respective points at the specified time, and multiplexed and delayed signals reach to a receiver under existence of additive white Gaussian noise $\{\eta_i\}$ with variance σ^2, and the desired $\ell' th$ signal is detected by a correlator with reference signal $\{\hat{b}^*_{\ell',i}\}$. If time delay and signal attenuation are ignored, the correlator output for the desired signal at shift $i' = 0$ is given by

$$\frac{K}{\hat{M}}\sum_{i=0}^{\hat{M}-1}(\hat{a}_{\ell',i} + \eta_i)\hat{b}^*_{\ell',i} = 1 + \frac{K}{\hat{M}}\sum_{i=0}^{\hat{M}-1}\eta_i\,\hat{b}^*_{\ell',i} \quad (65)$$

and the power of noise out is expressed as

$$N_{out} = (\frac{\sigma K}{\hat{M}})^2\sum_{i=0}^{\hat{M}-1}\left|\hat{b}^*_{\ell',i}\right|^2$$
$$= (\frac{\sigma K}{\hat{M}})^2\sum_{i=0}^{\hat{M}-1}(\sum_{i_0=0}^{M'-1}\sum_{i_2=\ell'}^{N+1+\ell'} b^*_{\ell',i_2-\ell'}\,\delta_{i-i_0,\mu i_2}a'_{\lambda,i_0})$$
$$\times(\sum_{i'_0=0}^{M'-1}\sum_{i'_2=\ell'}^{N+1+\ell'} b_{\ell',i'_2-\ell'}\,\delta_{i-i'_0,\mu i'_2}a'_{\lambda,i'_0})$$
$$\cong (\frac{\sigma K}{\hat{M}})^2 M'\sum_{i_2=0}^{N+1}\left|b^*_{\ell',i_2}\right|^2 = (\frac{\sigma K}{\hat{M}})^2 M'M|\varepsilon_N|\,\xi\ , \quad (66)$$

$$\xi = 4 + \frac{1}{M|\varepsilon_N|}\sum_{i_2=2}^{N-1}\left|b^*_{\ell',i_2}\right|^2 \quad (67)$$

where the absolute value of $b^*_{\ell',i_2}; i_2 = 0,1,N,N+1$ is $M|\varepsilon_N|$, as seen in the sequence $\{b^*_{\ell,i}\}$ of $N = 8$ and this nature holds for $N = 16, 32, \cdots$. For $N = 8$, referring to $\{b^*_{0,i}\}$ of (37), $\{b^*_{1,i}\}$ of (39), $\{b^*_{2,i}\}$ from (29) and $\{b^*_{3,i}\}$ from (31), we obtain

$$\xi = \begin{cases} 4 & ,\ell' = 0, \\ 4 + 6|w_1^2 - w_2^2|^2 & ,\ell' = 1,3, \\ 4 + 2|w_1^4 - w_2^4|^2 & ,\ell' = 2 \end{cases} \quad (68)$$

where $|w_1^2 - w_2^2| = |w^2 - w^{-2}| = 2\sin(2\beta_0/N)$ and $|w_1^4 - w_2^4| = 2\sin(4\beta_0/N)$.

The power of input signal $\{\hat{a}_{\ell',i}\}$ is given by

$$S_{in} = \frac{1}{M_{\hat{a}}}\sum_{i=1}^{M_{\hat{a}}}\left|\hat{a}_{\ell',i}\right|^2 = \frac{1}{M_{\hat{a}}}M'(N-1)M|\varepsilon_N| \quad (69)$$

where $M_{\hat{a}} = (N-1)\mu + M'$ is the length of $\{\hat{a}_{\ell,i}\}$.

Since peak power S_{out} of signal output is unity, the signal to noise power ratio of correlator output

$$SNR_{out} \cong \frac{M'M}{\sigma^2|\varepsilon_N|\xi} \quad (70)$$

and the signal to noise power ratio of correlator input

$$SNR_{in} \cong \frac{M'M}{\sigma^2\mu}|\varepsilon_N| \quad (71)$$

are obtained. Hence, the process gain is given by

$$G_p = SNR_{out}/SNR_{in} \cong \frac{\mu}{|\varepsilon_N|^2\xi}\ . \quad (72)$$

For $N = 8$ and $\beta_0 = \pi/8$, it follows $|\varepsilon_N|^2 = 0.293, \xi \cong 4, 4.23, 4.30$, $|\varepsilon_N|^2\xi \cong 1.17, 1.24, 1.26$ for $\ell' = 0,1(3),2$. By selecting the smaller β_0, $|\varepsilon_N|$ and $|\varepsilon_N|^2\xi$ gradually approach 1/2 and 1, respectively.

Similarly, the process gain on the real-valued orthogonal sequence pairs of (47) to (50) and corresponding pairs from (28) to (31) are given by

$$G_p \cong \frac{\mu}{|\varepsilon_N|^2\xi\zeta}\ . \quad (73)$$

where $SNR_{out} = \frac{M'M}{\sigma^2|\varepsilon_N|\xi}$, $\xi = \frac{1}{M|\varepsilon_N|}\sum_{i_2=0}^{N+1} b^2_{\ell,i_2}$, $SNR_{in} \cong \frac{M'M}{\sigma^2\mu}|\varepsilon_N|\zeta$ and $\zeta = \frac{1}{M|\varepsilon_N|}\sum_{i_1=1}^{N} a^2_{\ell,i_1}$.

When $N = 8, N\alpha = 0.1$, we obtain $\xi \cong 3.68, 3.87, 4.02, 4.28$ and $\zeta \cong 1.09, 1.03, 1.01, 0.94$ for $\ell' = 0,1,2,3$, respectively and $|\varepsilon_N| \cong 0.4975$ and $|\varepsilon_N|^2\xi\zeta \cong 0.993, 0.987, 1.00, 0.996$ for $\ell' = 0,1,2,3$, respectively and thus $G_p \cong \mu$.

From (72) and (73), the process gain is expressed as $G_p \cong \mu$, although strictly μ is replaced by $\mu + M'/(N-1)$. Decreasing of M' connects to decreasing of orthogonal area of (64). In the practical elastic wave probing, the great large process gain is desired for transmitting effective energy to the receiver. For example, sequence length is $M' = 10{,}000$ to $20{,}000$ and family number is 4 to 8 because the wave is emitted toward the wide angular area. If we select $\mu = M' = N' + 1$, the superposed amount of the inter-channel interference as $\hat{\rho}_{\ell,\ell',i'}$ of (62) may be negligible. In this case, we obtain $G_p \cong M'$ and the system is LCZ (low correlation zone) quasi-orthogonal one. In other case, if the wide ZCZ system is desired, we must select the sequence of length $M' \cong 1.5\mu$ though $G_p \cong \mu$.

D. Magnitude of transmitting sequence

One object of this paper is generation of suppressed magnitude transmitting sequence. In reference [7], a Huffman sequence with length 11822 and maximum magnitude 5.2 and rms value 1 is used, where power efficiency of transmitting amplifier is $1/5.2^2 \cong 1/27$ corresponding to the process gain loss. In this paper, rms values of complex-valued transmitting sequences $\{\hat{a}_{\ell,i}\}$ for $N' = \mu, 2\mu$ are approximately $\sqrt{M|\varepsilon_N|}, \sqrt{2M|\varepsilon_N|}$ from (69) and their magnitude are below $|a'_{\lambda,i}|_{max}$, $2|a'_{\lambda,i}|_{max}$ where $|a'_{\lambda,i}|_{max}$ is the maximum amplitude of the common real-valued sequence $\{a'_{\lambda,i}\}$, because $|a_{\ell,i_1}a'_{\lambda,i}| = \sqrt{M|\varepsilon_N|}|a'_{\lambda,i}|$ and $|a_{\ell,i_1}a'_{\lambda,i_2} + a_{\ell,i_3}a'_{\lambda,i_4}| \le \sqrt{M|\varepsilon_N|}(|a'_{\lambda,i_2}| + |a'_{\lambda,i_4}|)$ in the significant part to raise the magnitude of the convolved sequences. Thus, for $N' = \mu, 2\mu$, the magnitude of the normalized transmitting sequence are below $|a'_{\lambda,i}|_{max}$, $\sqrt{2}|a'_{\lambda,i}|_{max}$, and for $N' = 3\mu/2$, is estimated to be below $\sqrt{3/2}|a'_{\lambda,i}|_{max}$. In this paper, an example of the real-valued sequence of $|a'_{\lambda,i}|_{max} \cong 2.5$ is introduced.

VI. Signal Transmission and Processing

A. Transmission of complex-valued sequences

The transmitting signal is generated by weighting real part $\hat{r}_{\ell,i}$ and imaginary part $\hat{s}_{\ell,i}$ of $\{\hat{a}_{\ell,i}\}(\ell = 1,\cdots,\frac{N}{2} - 1)$ on cosine and sine carriers, respectively and adding them, and the signal is converted to physical wave and transmitted toward the path at the respective point. Multiplexed and delayed waves reach a receiver at the other distant point and are converted to electric signal by sensor and demodulated to cosine and sine component and reconstructed to complex-valued signal and correlated with the desired sequence $\{\hat{b}^*_{\ell',i}\}(\ell' = 1,\cdots,\frac{N}{2} - 1)$. The $\ell' th$ received signal with time delay τ and angular frequency ω_0 of carrier and chip time interval T_c is given by

$$\hat{a}_{\ell'}(t-\tau) = \sum_{i=1}^{M_{\hat{a}}}\{\hat{r}_{\ell',i}cos\omega_0(t-\tau) + \hat{s}_{\ell',i}sin\omega_0(t-\tau)\} \times g(t-\tau-iT_c) \quad (74)$$

where $g(t)$ is rectangular chip waveform with height 1 and duration T_c. The complex-valued signal reconstructed by in-phase and quadrature-phase components is given by

$$\hat{a}_{\ell'}(t-\tau) = \frac{1}{2}\sum_{i=1}^{M_{\hat{a}}} \hat{a}_{\ell',i}e^{j\omega_0\tau}\, g(t-\tau-iT_c) \quad (75)$$

and the desired output signal of correlator is given by

$$\begin{aligned}\hat{\rho}_{\ell',\ell'}(t') &= \frac{1}{\tilde{M}T_c}\int_0^{(\tilde{M}-1)T_c} \hat{a}_{\ell'}(t-\tau)\,\hat{b}^*_{\ell'}(t-t')dt \\ &= \frac{e^{j\omega_0\tau}}{2}\sum_{i'=-(\tilde{M}-1)}^{\tilde{M}-1}\hat{\rho}_{\ell',\ell',i'}\,h(t'-\tau-i'T_C) \\ &= \frac{e^{j\omega_0\tau}}{2}h(t'-\tau),\ i'=0\,, \end{aligned} \quad (76)$$

where the $\ell' th$ reference signal to correlator is given by

$$\hat{b}^*_{\ell'}(\mathrm{t}) = \sum_{i=0}^{M_{\hat{b}}-1} \hat{b}^*_{\ell',i}g(t-iT_C) \quad (77)$$

and $M_{\hat{b}} = (N+1)\mu + M'$ is the length of $\hat{b}^*_{\ell',i}$ and the autocorrelation function of $g(t)$ is given by

$$h(t') = \frac{1}{T_c}\int_{-\infty}^{+\infty} g(t)g(t-t')dt \quad (78)$$

which is triangular pulse of height 1 and base $2T_c$.The absolute value of (76) gives the intensity of response $h(t'-\tau)$ though negative or positive reflection cannot be discriminated.

B. Transmission of real-valued sequences

The signals constructed from real-valued orthogonal sequence pairs do not always accompany sinusoidal carrier. In case of using cosine carrier, imaginary parts of $\{\hat{a}_{\ell',i}\}$ and $\{\hat{b}^*_{\ell',i}\}$ disappear, and
in-phase correlation output

$$I[\hat{\rho}_{\ell',\ell'}(t')] = \frac{1}{2}(cos\omega_0\tau)h(t'-\tau),\ i'=0\,, \quad (79)$$

and quadrature phase correlation output

$$Q[\hat{\rho}_{\ell',\ell'}(t')] = \frac{1}{2}(sin\omega_0\tau)h(t'-\tau),\ i'=0 \quad (80)$$

and response intensity

$$\sqrt{I^2+Q^2} = h(t'-\tau)/2,\ i'=0 \quad (81)$$

are obtained [7].

In case of using no carrier, the correlator output

$$\hat{\rho}_{\ell',\ell'}(t') = h(t'-\tau),\ i'=0 \quad (82)$$

is obtained and is multiplied by $+1$or -1 sign corresponding to $+1$ or -1 reflex coefficient through transmission path.

VII. Conclusion

For underground multiplexing coded sonar, finite-length orthogonal ZCZ sequence pairs of complex values or real values are proposed. Orthogonal sequence pair is lengthened by $(\mu-1)$zero value insertion and convolved with a real-valued common sequence of length $M' = N'+1$. Selection of $N' = 3\mu/2$ gives wide ZCZ and process gain μ. The magnitudes of transmitting sequences are estimated to be below$\sqrt{3/2}|a'_{\lambda,i}|_{max}$for complex-valued sequence pairs. Selection of $M' = \mu$ gives wide LCZ and process gain M'.These selections follow applications using complex- or real-valued pairs. The merits of this sequence set are to have the impulsive auto-equivalent correlation function detect its reaching point and to transmit signals through path suitable to narrow carrier band or base band. Those are important factors for geophysical measurements.

The application to practical instrumentation is a goal in the future, where we can use such technique that the value of real or imaginary part is converted to $+1$ and -1 values by high-efficient encoding method [9].

References

[1] C. Bagaini, M. Daly and I. Moore, "The acquisition and processing of dithered slip-sweep vibroseis data," *Geophysical Prospecting*, vol.60, pp.618-639, Jul. 2012

[2] J. Li and P. Stoica (eds.), *MIMO Radar signal processing,* J.Wiley & Sons,USA:2009.

[3] A.Rathinakumar and A.K.Chaturvedi, "Complete mutually orthogonal Golay complementary sets from Reed-Muller codes," *IEEE Trans. Inf.Theory*,vol.54, no.3,pp.1339-1346,Mar.2008.

[4] D.A. Huffman, "The generation of impulse-equivalent pulse train," *IEEE Trans. Inf. Theory*, vol.IT-8, pp. s10-s16, Sep. 1962.

[5] Y.Tanada, "Orthogonal set of Huffman sequences and its application to suppressed-interference quasi-synchronous CDMA system," *IEICE Trans. Fundamentals Electron..Commun.Comput.Sci.,* vol. E89-A, no.9, pp.2283-2291, Sep.2006.

[6] Y.Tanada, K.Sato, "Long Huffman sequence derived from even functional quadratic residues," *in Proc. IEEE 6th Int. Workshop Signal Design Appl. Commun.(IWSDA)*, Oct. 2013, pp.56-59.

[7] Y.Tanada, M.Iguchi, K.Yamamoto, H.Nakamichi and Y.Morita, "Elastic wave propagation characteristics of Sakurajima Volcano underground layer using coded sonar," *in Proc.SICE 35th Sensing Forum*, Aug.2018, pp.63-67.

[8] Y.Tanada, M. Iguchi and T.Tsutsui, "Construction of finite-length sequences for underground multiplexing coded sonar, *IEICE Tech. Report, CS2019-19, pp.125-130, Feb.2020.*

[9] P.Shao and Y.Tanada, "Improved Two-Valued and Three-Valued Integrand Codes for Real-Valued Self-Orthogonal Finite-Length Sequence," *in Proc.IEEE Int.Symp.Intelligient Signal Processing and Commun. Systems (ISPACS)*, Dec.2006, pp.907-910.