Performance Analysis of QC-LDPC codes constructed by using Golomb rulers

Daekyeong Kim, Inseon Kim, Hyunwoo Cho, Hyojeong Choi and Hong-Yeop Song
School of Electrical and Electronic Engineering
Yonsei University
Seoul, Korea
{daky33, is.kim, hyunwoo.cho, hjchoi3022, hysong}@yonsei.ac.kr

***Abstract*—In this paper, we analyzes performance of girth-8 regular QC-LDPC codes constructed using Golomb ruler. We conducted simulations to measure FER performance of QC-LDPC codes constructed by changing the last mark of some optimal Golomb ruler and found the value of the mark that shows the best performance.**

***Index Terms*—Golomb ruler, QC-LDPC codes, FER**

I. Introduction

Channel codes used in Global Navigation Satellite System are required to have robust error correction capability with length around 1000. With short length, Low-Density Parity-Check(LDPC) codes from algebraic construction show better performances than those from random construction in general. Quasi-Cyclic(QC)-LDPC codes can be constructed by using algebraic structure.

In [1], short length QC-LDPC codes are constructed by using Golomb ruler. Varying the last mark of optimal Golomb ruler, constructed QC-LDPC codes from new Golomb rulers show good performances. Howevere, it is not found in [1] that which value of last mark make QC-LDPC code have the best performance.

In this paper, we vary last mark of optimal Golomb ruler within whole possible range suggested in [1] and check performances of QC-LDPC codes constructed from them to determine best choice of last mark.

II. Preliminary

A Golomb ruler is a set of n integers $g_1, g_2, ..., g_n$ in ascending order with distinct $g_i - g_j$ for every $i < j$ and a Golomb ruler that has n marks is called n-mark Golomb ruler [2]. The length L of a n-mark Golomb ruler is the maximum distance $g_n - g_1$ between two marks and an optimal n-mark Golomb ruler is an n-mark Golomb ruler that has the minimum length L possible [2], [3]. A some new n-mark Golomb ruler can be made by changing only last mark g_n of an optimal n-mark Golomb ruler as g'_n if

$$g'_n > 2g_{n-1} \tag{1}$$

One way of constructing a QC-LDPC codes is using a multiplication table. By considering given multiplication table as exponent matrix, a linear code which is defined by a parity check matrix H constructed by substituting each elements of exponent matrix by matrices obtained by circularly-shifting the $P \times P$ identity matrix I_P by the value of each elements is a QC-LDPC code [1], [4]–[6].A QC-LDPC code has neither 4-cycle nor 6-cycle in their Tanner graph of parity check matrix if it is constructed by using multiplication table whose top row is Golomb ruler of length L and left most column is 1, 2, 3 and if

$$2L < P \tag{2}$$

where P is the size of identity matrix I_P used in construction process [1], [5].

For a given optimal n-mark Golomb ruler and CPM size P, one can derive a condition of last mark X for non-existence of 4-cycle and 6-cycle from (1) and (2) :

$$2g_{n-1} < X < P/2 \tag{3}$$

III. Simulation

We constructed QC-LDPC codes by changing 6-th mark of optimal 6-mark Golomb ruler (0,1,8,12,14,17) into values from 29 to 74 for P = 150 and from 29 to 99 for P = 200 correspond to (3). And by Monte-Carlo simulation, we measured frame error rate (FER) of QC-LDPC codes. Assuming AWGN channel and BPSK modulation, we conducted simulation using sum-product decoding of the maximum iteration number 50 and check the FER at E_b/N_0 = 0, 0.5, 1, ... dB until curves cross the FER 10^{-3} line as shown in Fig.1 and Fig.2.

We checked that 59 was the best 4 marks of showing performance about 2.61 dB for P = 150, N = 900 and 97 was the best 4 marks of showing performance about 2.54 dB for P = 200, N = 1200. The two groups of performance curve in each Fig.1 and Fig.2 showed clear difference. The group B is corresponding to the performance curves of changing the last mark as 50, 51, 58, 62, 64 and the group A is the other cases in Fig.1. The group C is corresponding to the performance curves of changing the last mark as 50, 51, 58, 62, 64 and the group D is the other cases in Fig.2.

We listed values of last mark in ascending order of E_b/N_0 at FER 10^{-3}. Table I are the lists of 6-th mark of Golomb rulers used for each case of group A, B, C, D in ascending order of the E_b/N_0 values at FER 10^{-3}. And, we made scatter plot of E_b/N_0 at FER 10^{-3} versus value of last

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2020R1A2C2011969).

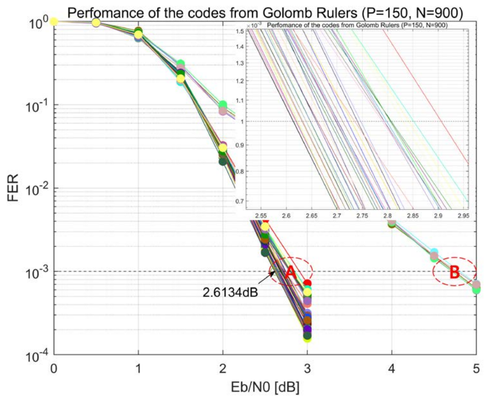

Fig. 1. Performance of the codes from Golomb ruler (P=150, N=900).

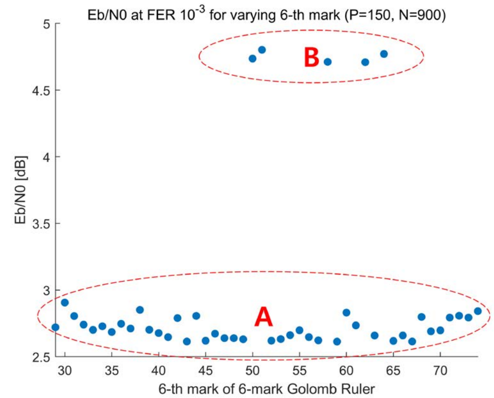

Fig. 3. E_b/N_0 at FER 10^{-3} for varying 6-th mark (P=150, N=900).

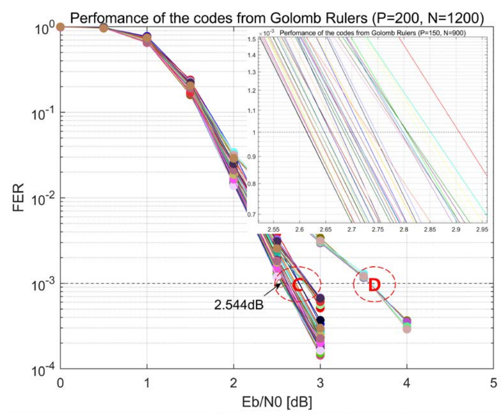

Fig. 2. Performance of the codes from Golomb ruler (P=200, N=1200).

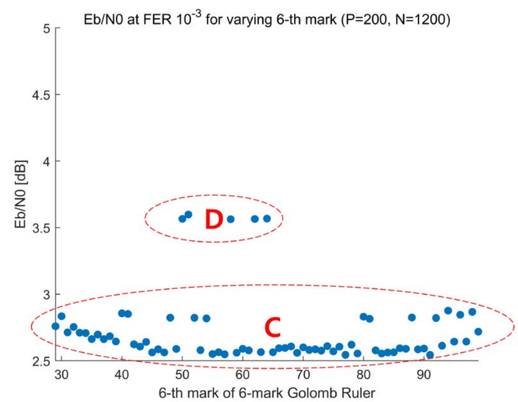

Fig. 4. E_b/N_0 at FER 10^{-3} for varying 6-th mark (P=200, N=1200).

mark since they are two variables of our interest. The E_b/N_0 values at FER 10^{-3} according to the 6-th mark of Golomb rulers are shown in Fig.3 and Fig.4. Performances for group A are from about 2.61 dB to 2.91 dB for group A. Performances for group B are from about 4.71 dB to 4.8 dB. Performances for group C are from about 2.54 dB to 2.88 dB. Performances for group D are from about 3.56 dB to 3.6 dB.

IV. Conclusion

In this paper, simulation for performance of QC-LDPC codes constructed by changing only last mark of an optimal Golomb rulers in the range of 4-cycle and 6-cycle free condition was conducted. From the simulation result, we checked the best choice of last mark in construction is 59 and 91 for QC-LDPC code with length 600 and 1200 respectively.

TABLE I
6-th marks in ascending order of E_b/N_0 at FER 10^{-3}

Group	6-th mark
A	59,43,67,65,45,52,57,49,53,48,47,41,56, 63,66,54,46,40,35,69,70,55,33,39,37,29,34, 61,32,36,42,71,73,68,31,44,72,60,74,38,30
B	62,58,50,64,51
C	91,77,57,55,83,79,69,59,84,47,45,85,65, 56,63,75,73,61,82,53,71,89,72,46,49,87, 60,90,86,66,67,70,76,43,68,74,93,78,42, 44,95,97,39,37,35,38,36,34,33,31,99,32, 29,81,54,92,52,48,88,80,30,96,41,40,98,94
D	58,62,50,64,51

Acknowledgment

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT) (No. 2020R1A2C2011969).

References

[1] I. Kim and H.-Y. Song, “A construction for girth-8 QC-LDPC codes using Golomb rulers,” accepted for publication by Electronics Letters

[2] A. K. Dewdney, “Computer recreations,” Scientific American, vol. 253, no. 6, pp.16-29, Dec. 1985.

[3] G. S. Bloom and S. W. Golomb, “Applications of numbered undirected graphs,” Proceedings of the IEEE, vol. 65, no. 4, pp.562-570, Apr. 1977.

[4] M. P. C. Fossorier, “Quasicyclic Low-density Parity-check Codes from Circulant Permutation Matrices,” IEEE Trans. Info. Theory, vol. 50, pp. 1788-1793, July 2004.

[5] I. Kim and H.-Y. Song, “Some New Constructions of Girth-8 QC-LDPC Codes for Future GNSS,” IEEE Communications Letters, vol. 25, no. 12, pp. 3780-3784, Dec. 2021.

[6] R. M. Tanner, D. Sridhara, A. Sridharan, T. E. Fuja and D. J. Costello, “LDPC Block and Convolutional Codes Based on Circulant Matrices,” IEEE Trans. on Info. Theory, vol. 50, no. 12, pp. 2966–2984, Dec. 2004.