

# Tolerable Visuo-Tactile Inconsistencies in Periodic and Random Grating Textures on Surface Texture Displays

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**Abstract**—In surface tactile displays, consistency between visual and tactile feedback is generally regarded as critical. However, subtle inconsistencies between these modalities often go unnoticed by users. This study examines the tolerance for visuo-tactile gaps in texture perception, specifically focusing on grating scales, using a surface friction display. To encompass a wide range of textural stimuli, we employed both periodic and random gratings. Participants adjusted the tactile grating scales (average wavelength) to match visual grating scales, enabling the identification of 84% discrimination limens for two reference mean wavelengths: 1.0 mm and 2.0 mm.

The minimally allowable inconsistencies, defined as the 84% discrimination limens, were measured as 0.315 mm and 0.422 mm for periodic gratings and 0.256 mm and 0.327 mm for random gratings at the 1.0 mm and 2.0 mm wavelength levels, respectively. Results showed that random gratings exhibited significantly narrower tolerance ranges compared to periodic gratings. These findings are valuable for the design of haptic content, as they inform the hardware performance specifications, particularly the precision required in sensing finger movements on touch panels.

**Index Terms**—electrostatic texture display, surface display, discrimination, gratings

## I. INTRODUCTION

Touch panels are among the most popular human-computer interfaces. Hence, surface tactile displays, a haptic feedback technique for touch panels, have been extensively studied by researchers (e.g., [1]–[4]). Implementing tactile feedback functions on touch panels enables a variety of applications, including drawing and designing, simulated mechanical buttons, and slider- or knob-like interfaces [5]–[12]. Additionally, such feedback enhances the entertainment value [13], [14] and usability [10]–[12], [15] of computer applications.

For the development of effective interfaces, it is crucial that visual and tactile content spatiotemporally align; however, a certain degree of inconsistency is permissible in texture rendering [16], [17]. Yamaguchi et al. [16] compared the spatial wavelengths of grating scales displayed on an LCD screen with those of physical grating scales made of resin using a modified up-down method and calculated discrimination thresholds. This thresholds can be considered as acceptable difference between visual and tactile stimuli. Meanwhile, Kurita et al. [17] investigated discrimination thresholds for wavelengths between visual and tactile grating scales using a

friction-variable surface texture display and the psychophysical method of adjustment. Their findings [16], [17] revealed that the wavelengths of visual and tactile grating scales must differ by approximately 20% or more before users begin to perceive inconsistencies [16], [17].

In this study, we also investigated visuo-tactile inconsistencies using grating scales, building on previously established methods as described above. Our research complements prior studies in two key aspects. First, while previous studies [16], [17] focused on grating scales with periodic surface patterns, we expanded the scope or domain to include those with random patterns. Random gratings often serve as substitutes for natural textures because they encompass a broader range of frequencies compared to periodic textures [18]. Thus, discrimination thresholds for visuo-tactile inconsistencies may differ between random and periodic gratings.

Second, Yamaguchi et al. [16] compared grating scales displayed on an LCD panel with physical grating scales made of resin. In contrast, our study examined the allowable visuo-tactile inconsistency in scenarios where both visual and tactile textures were presented on the same surface using a friction-variable surface texture display. This approach is particularly relevant for the practical use of surface texture displays. Moreover, the grating scales generated by friction-variable surface texture displays exhibit differences in perceived roughness compared to physical grating scales [19], potentially due to the lack of skin-penetration effect into inter-ridge grooves [20], [21]. Thus, it is crucial to investigate visuo-tactile inconsistencies using textures presented on touch panels to ensure practical applicability.

This study aims to determine the allowable inconsistency between visual and tactile texture stimuli for surface texture displays that deliver electrostatic friction. Identifying the acceptable level of inconsistency between visual and tactile stimuli will assist developers in creating cost-effective haptic content. Bocheureau et al. [22] suggested that high-precision or low-latency measurement of finger motion may not be necessary for texture displays. This assertion is based on the relatively low discrimination ability of humans for the frequency of vibratory tactile stimuli, which is roughly 20% in Weber fraction across a wide frequency band [23]–[25]. If this limited discrimination ability is the primary reason why inconsistencies between visual and tactile grating scales are

This research was in part funded by MEXT Kakenhi (24K03019 and 23H04360).

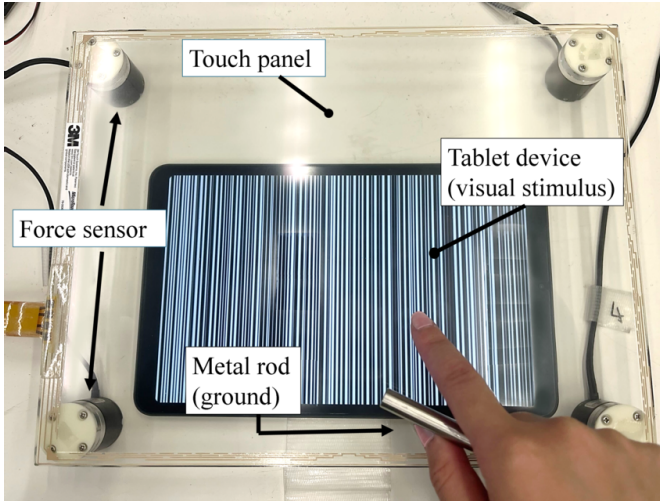


Fig. 1: Electrostatic friction display based on a capacity-type touch panel. Still images of the grating textures were shown on a tablet PC beneath the touch panel.

permissible, clarifying their tolerable discrepancy will contribute to the design of more cost-effective haptic interfaces.

## II. METHODS

### A. Apparatus: Electrostatic Tactile Texture Display

An electrostatic tactile display shown in Fig. 1 was used in the study. The same apparatus was also used in [26], [27]. Voltages of up to  $\pm 20$  V were applied to the indium tin oxide touch panel (3M Touch Systems, Inc., MN), over which the participant directly slid their finger. The voltage was adjusted for individuals as described in Section II-B2. To stabilize the stimuli, participants gripped a grounded metal rod.

The electric charges between the finger and touch panel produced an adhesive frictional force, leading to the increase in friction. The intensity of this force is controlled by changing the level of applied voltage. A force sensor was placed beneath each corner of the panel and they were used to calculate the center of the load on the panel, corresponding to the finger position. A data acquisition device (PEX-361216, Interface Corp., Japan) was used to control the output voltage to the touch panel and calculate the finger position at the control frequency of 2 kHz. The tablet used in the experiment had a display size of 8.4 inches and a resolution of  $1920 \times 1200$  pixels.

### B. Stimuli

1) *Periodic and Random Grating Scales*: The grating scale function  $g(x)$ , a binary function that takes either 0 (groove) or 1 (ridge), was defined as follows.

For periodic grating scales,  $g(x)$  was determined by:

$$g(x) = \begin{cases} 0 & (\sin \frac{2\pi x}{\lambda} > 0), \\ 1 & (\sin \frac{2\pi x}{\lambda} \leq 0), \end{cases} \quad (1)$$

where  $x$  ( $\geq 0$ ) represents the mediolateral position of the finger on the touch panel, and  $\lambda$  (mm) denotes the wavelength

of the gratings. In the experiments, periodic gratings with  $\lambda$  ranging from 0.5 mm to 5.0 mm were presented.

For random grating scales,  $g(x)$  was determined using the following equations, where  $\lambda$  represents the average wavelength:

$$g(x) = \begin{cases} 0 & (x \in G_i \mid i = 1, 2, \dots), \\ 1 & (x \in R_i \mid i = 1, 2, \dots), \end{cases} \quad (2)$$

$$G_i = [g_{i-1}, r_i], \quad (3)$$

$$R_i = (r_i, g_i), \quad (4)$$

$$g_0 = 0, \quad (5)$$

$$r_i = g_{i-1} + b_i, \quad (6)$$

$$g_i = r_i + d_i, \quad (7)$$

$$B \sim U(0.2, \lambda - 0.2), \quad (8)$$

$$D \sim U(0.2, \lambda - 0.2), \quad (9)$$

where  $b_i$  and  $d_i$  are the  $i$ -th realizations of the uniform random variables  $B$  and  $D$ , respectively. Here,  $b_i$  and  $d_i$  represent the local groove and ridge lengths, with minimum and maximum values of 0.2 mm and  $\lambda - 0.2$  mm, respectively. Their mean values are  $\lambda/2$ , resulting in an expected wavelength (sum of the local groove and ridge lengths) equal to  $\lambda$ . The minimum value of 0.2 mm was determined based on the control frequency of the tactile texture display.

2) *Tactile Stimuli*: The tactile textures were presented using the electrostatic display, where the applied voltage  $V_e(t)$  was determined as:

$$V_e(t) = Ag(x(t)) \quad (10)$$

where  $A$  is the voltage value determined for individuals. The voltage was set to a high value when the grating scale function  $g(x)$  was 1 (ridge) and to zero when  $g(x)$  was 0 (groove). The polarity of  $V_e(t)$  was alternated at a frequency of 2 kHz to ensure effective control of the adhesion force.

The relationship between the shear frictional force  $F_e(t)$  and the applied voltage  $V_e(t)$  was governed by the law of electrostatic force and Coulomb friction, as follows [28]:

$$F_e(t) = \mu\{W + kV_e^2(t)\}, \quad (11)$$

where  $\mu$ ,  $W$ , and  $k$  are the coefficient of friction, the load exerted by the finger, and a constant related to electrostatic force, respectively. The constant  $k$  is primarily influenced by the dielectric constant and the thickness of the finger's superficial skin, as well as the insulating material of the touch panel.

3) *Visual Stimuli*: The visual grating scales consisted of alternating white and black bars, as shown in Fig. 2. They depict periodic and random gratings with (average) wavelengths of  $\lambda = 1.0$  mm and 2.0 mm, where the ratio of the white and black areas were equal.

### C. Participants

Fifteen university students in their 20s participated in the study after providing written informed consent. All participants were unaware of the study's objectives. Thirteen participants completed the experiment with the periodic stimuli, and

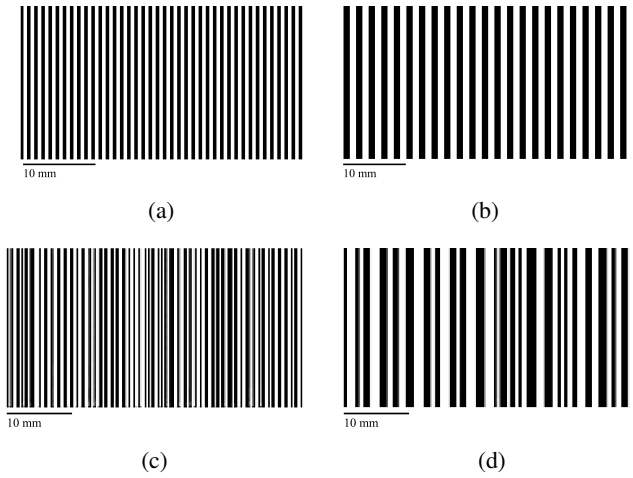


Fig. 2: Examples of grating scales with wavelengths of 1.0 mm (a), 2.0 mm (b), random grating scales with average wavelengths of 1.0 mm (c) and 2.0 mm (d).

eleven participated in the experiment with the random stimuli. The two types of stimuli were tested on different days, and some participants failed to join the two experiments because of the difficulties in personal scheduling.

#### D. Ethical Statement

The protocol for this study was approved by the Institutional Review Board of Hino Campus, Tokyo Metropolitan University (approval number: R6-009).

#### E. Procedures

Participants wiped their hands with ethanol-soaked tissues to keep the surfaces of their fingers reasonably dry. They used their dominant hand. In the training session, each participant adjusted the intensity of the electrostatic stimuli such that they could clearly feel the tactile gratings. During this session, they were presented with both visual and tactile gratings whose surface wavelengths matched. The  $\lambda$  values of 1.0 mm and 2.0 mm were used for this task.

In the main session, we employed the psychophysical method of adjustment due to its relatively low experimental cost. The participant slid their finger on the panel and compared the wavelengths of the visual and tactile grating scales. The wavelength of the tactile scale was variable, starting with a random value ranging from 0.5 mm to 2.5 mm at the beginning of each trial. The participant adjusted the wavelength of the tactile scale by 0.1 mm increments or decrements by using a keyboard until the tactile scale felt equal to the visual scale. Participants were allowed to change the wavelength by more than 0.1 mm at a time. Each participant repeated this trial 10 times for each of the two visual wavelength levels, with no time limit and no limit on the number of adjustments. For those experimenting with the two types of grating scales, the testing order was randomized.

A previous study reported a pooled standard deviation of 0.33 mm for a wavelength of 1.0 mm [17]. Based on this, the

number of trials was set to 10 to keep the standard error close to the display's minimum adjustable step of 0.1 mm.

Some participants completed experiments with both periodic and random stimuli on the same day, while others did so on separate days; the experimental order was counterbalanced. Each session took approximately 40 min to complete.

#### F. Data Analysis

Some participants were unable to adjust the tactile wavelength to match the visual stimulus, potentially because they did not perceive the frictional stimuli presented by the display. This issue could arise due to changes in the condition of their finger pads, such as sweating during the experiment. These participants were excluded based on the following criteria.

For each participant, the mean  $\pm 2.26$  times the standard deviation of their responses was calculated, where 2.26 is the  $t$ -value corresponding to a cumulative distribution function value of 0.975 with 9 degrees of freedom. Only participants whose visual stimulus wavelengths (i.e., 1.0 mm or 2.0 mm) fell within this range were included in the statistical analysis.

The Shapiro-Wilk test was applied to assess the normality of participants' responses. Previous studies [29], [30] have not clearly established whether detection thresholds should be calculated individually or by pooling data across participants. In our study, we aimed to verify normality at the individual level, as the thresholds were computed separately for each participant and then averaged. However, due to the limited sample size of 10 trials per participant—which is generally insufficient for robust hypothesis testing—we instead assessed normality using pooled responses across participants.

Therefore, we assessed normality using pooled responses from all participants for each condition, defined as the combination of the grating scale type and the average  $\lambda$  value. To facilitate pooling, each participant's response values were adjusted to have a mean of zero. Further, samples outside the mean  $\pm 2$  standard deviation were excluded as outliers.

The  $p$ -values exceeded 0.05 in all conditions, and thus, the null hypothesis of normality was not rejected. Accordingly, the obtained samples were treated as following a normal distribution.

The 84% discrimination threshold was calculated from the data obtained using the method of adjustment. In this study, the 84% threshold was defined as the difference between the test and reference stimuli at which participants judged the test stimulus to be greater than the reference in 84% of trials. Assuming normality in participants' responses, the 16% and 84% thresholds are symmetric about the mean or the point of subjective equality. These thresholds were calculated using the standard deviation of the adjusted stimulus values. This method offers a simplified approach to estimating discrimination thresholds [29], [30], although the method of constant stimuli more closely conforms to the probabilistic definition of a threshold.

Given the limited number of samples per participant, two calculation methods were used.

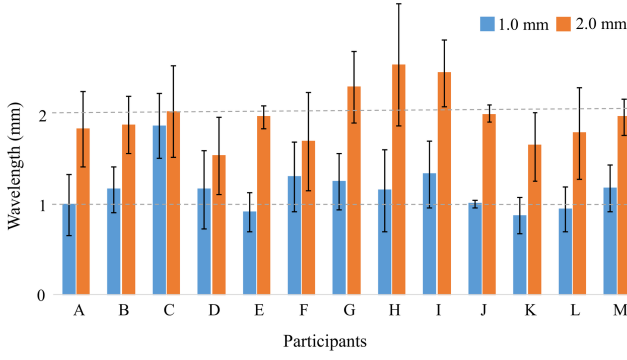


Fig. 3: Mean values and 95% confidence intervals of all participants' responses to the periodic stimuli. The blue and orange bars represent the means for visual stimulus wavelengths of  $\lambda = 1.0$  mm and  $\lambda = 2.0$  mm, respectively.

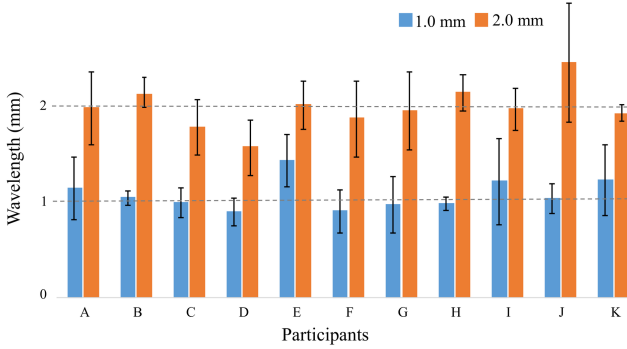


Fig. 4: Means and 95% confidence intervals of all participants' responses to the random stimuli.

The first method was based on the pooled standard deviation for each condition. In this approach, the discrimination threshold is not the average of those calculated for individual participants; instead, a single value derived from all participants' pooled data is discussed.

The second method involved calculating the thresholds for individual participants, followed by analyzing their mean. While we speculate that this approach is preferable if the normality of the sample distribution is confirmed for each individual, this assumption did not hold in our study.

### III. RESULTS

Fig. 3 shows the mean values and confidence intervals of individual responses across 10 trials for the periodic grating scales. For the 1.0 mm periodic visual stimulus, 12 out of 13 participants met the inclusion criterion, with participant C excluded. For the 2.0 mm periodic visual stimulus, 10 out of 13 participants met the criterion, with participants D, I, and K excluded.

Fig. 4 shows the mean values and confidence intervals of participants' responses for the random grating scales. For the 1.0 mm random visual stimulus, 10 out of 11 participants met the inclusion criterion, with participant E excluded. Similarly,

TABLE I: 84% discrimination thresholds calculated as the pooled standard deviations and 95% confidence interval.

Scale type	Wavelength $\lambda$ (mm)	84% discrimination thresholds (mm)	95% confidence interval (mm)
Periodic	1.0	0.315	0.278–0.364
Periodic	2.0	0.412	0.340–0.445
Random	1.0	0.256	0.224–0.300
Random	2.0	0.327	0.286–0.383

TABLE II:  $F$ -tests of variances to compare the periodic and random grating scales.

Wavelength (mm)	Periodic (mm)	Random (mm)	$p$ value	$F$ statistics
1.0	0.315	0.256	0.020	1.52
2.0	0.412	0.327	0.008	1.67

TABLE III: Means and standard deviations of the 84% thresholds among individual participants. The 84% thresholds were separately calculated for individuals.

Scale type	Wavelength $\lambda$ (mm)	Mean discrimination threshold (mm)	95% confidence interval (mm)
Periodic	1.0	0.295	0.262–0.328
Periodic	2.0	0.345	0.289–0.401
Random	1.0	0.225	0.185–0.265
Random	2.0	0.288	0.236–0.340

for the 2.0 mm random visual stimulus, 10 participants met the criterion, with participant D excluded.

Table I shows the 84% discrimination thresholds calculated as the pooled standard deviations and 95% confidence intervals for 1.0 mm and 2.0 mm for each type of grating scale. For periodic scales, the 84% discrimination threshold for wavelengths 1.0 mm was 0.315 mm, and the confidence interval was 0.278–0.364 mm. The threshold for wavelengths 2.0 mm was 0.412 mm, and the confidence interval was 0.340–0.445 mm. For random scales, the threshold for the average wavelength of 1.0 mm was 0.256 mm, and the confidence interval was 0.224–0.300 mm. The threshold for the average wavelength of 2.0 mm was 0.327 mm, and the confidence interval was 0.286–0.383 mm.

Table II shows the results of statistic comparison between the periodic and random grating scales.  $F$ -tests on the pooled variances indicated that the thresholds for the periodic gratings were greater than those for the random gratings for the reference wavelength of 1.0 mm ( $F(108, 90) = 1.52$ ,  $p = 0.020$ ) and 2.0 mm ( $F(90, 90) = 1.67$ ,  $p = 0.008$ ). Further, the thresholds for 2.0 mm were greater than those for 1.0 mm for the periodic scale ( $F(90, 108) = 1.79$ ,  $p = 0.002$ ) and random scale ( $F(90, 90) = 1.63$ ,  $p = 0.010$ ).

Table III shows the means and standard deviations calculated from the discrimination thresholds for the individual participants. These values tend to be smaller than those based on the pooled standard deviations shown in Table I.

### IV. DISCUSSIONS

Tolerances for periodic gratings were 0.315 mm and 0.412 mm for  $\lambda = 1$  mm and 2 mm, and 0.256 mm and 0.327 mm for the random gratings, respectively. The values

for the periodic stimuli were greater than those for the random stimuli. Although we expected that the values for the random stimuli would be larger (more acceptable) than those for the periodic stimuli because of the ambiguity of peak vibratory frequency determined by the ratio of the sliding speed and average surface wavelength. However, the results contradicted this expectation.

A potential reason for the participants' higher sensitivity to visuo-tactile inconsistencies in random textures may be related to the high-order statistics inherent in random textures [31], [32]. While surface wavelength was the primary parameter manipulated in this study to define the features of periodic gratings, high-order information—such as variations in local wavelength and phase spectra—could have served as cues for detecting visuo-tactile inconsistencies of random gratings. In other words, although the parameter we controlled was the mean wavelength, other higher-order features varied simultaneously, which participants might have detected. For example, the variance of the  $\lambda$  value is 0 for periodic scales, whereas it is a function of  $\lambda^2$  for random scales. In the latter case, the mean and variance of  $\lambda$  vary together, providing more cues about  $\lambda$ . It remains unclear whether humans actively utilize high-order information in the discrimination of random tactile textures [31], [33], but this possibility should be explored in future research.

Also, for both types of grating scales, the discrimination thresholds were greater for 2 mm than those for 1 mm. Considering the constancy of Weber's fraction, these results are understandable and in agreement with earlier studies [16], [17]. Nonetheless, the Weber's fractions were not constant in our study. For the grating scale of 1 mm, the Weber's fractions were 31.5% and 25.6% for the periodic and random scales, respectively. For the grating scale of 2 mm, these values were 20.6% and 16.3%, respectively. The fractions were smaller for larger wavelengths. A similar trend was also confirmed by Yamaguchi et al. [16]. They reported the Weber's fraction of approximately 55% and 35%, for the wavelengths of 1 mm and 2 mm, respectively. It is noted that these values are calculated using 75% discrimination thresholds. This inequality of Weber's fraction can result from the perceptual nature of simulated tactile gratings. Otake et al. [34] reported that the fraction for the grating with smaller wavelength was greater than that with greater wavelength when they were presented by using a surface texture display.

A previous study has reported that roughness discrimination sensitivity improves for surface wavelengths larger than 2.0 mm [35]. In their study, participants made direct contact with roughness samples using the bare fingers, and the researchers suggested that the spatial distribution of SAI unit activity contributed to the enhanced sensitivity. The relatively small Weber's fraction observed for the 2.0 mm wavelength in our study may be discussed in relation to these findings. However, it should be noted that the surface of the texture display used in our study was flat, and thus the spatial activation patterns of SAI units expected with real roughness samples are unlikely to occur.

The discrimination threshold for tactile perception of grating scales may not strictly follow Weber's law, as similar deviations have been reported in previous studies [16], [34]. One proposed direction for future research is to investigate discrimination thresholds over a broader range of wavelengths, including 3.0 mm, to reexamine the behavior of Weber's fraction. This would help clarify whether the small Weber's fraction observed at 2.0 mm is a specific phenomenon or part of a monotonic decrease with increasing wavelength.

It is intriguing to compare the results of this study with those reported by Yamaguchi et al. [16]. The 75% discrimination thresholds reported by Yamaguchi et al. were 14%–48% larger than those calculated in this study. If this difference is inherent, it could be attributed to the following factors: 1) differences in tactile stimuli, 2) differences in psychophysical methods, and 3) differences in visual information.

First, regarding differences in tactile stimuli, Yamaguchi et al. used physical grating scales made of resin, whereas we employed a tactile display. Discrimination thresholds for physical grating scales are likely smaller than those for virtual gratings, which only replicate some characteristics of real stimuli. Thus, this factor does not adequately explain the differences between the two studies.

Second, Yamaguchi et al. used the modified up-down method [36] to determine the 75% discrimination thresholds, whereas we employed the method of adjustment. It is known that different psychophysical measurement methods can yield different threshold values [37], [38]. In general, the method of adjustment is considered relatively less accurate compared to other psychophysical methods [39].

Third, visual information differences must be considered. In our experiment, participants could see their fingers sliding over the visual texture, whereas in Yamaguchi et al.'s study, the evaluator's hand was hidden beneath the LCD. The size of one's own fingers can serve as a cue for judging the dimensions of the visible grating. Thus, it is possible that participants in our study were able to visually assess the grating scale dimensions more accurately, making them more sensitive to visuo-tactile discrepancies. To confirm this hypothesis, future experiments could replicate our conditions while obscuring the participants' hands from view.

One limitation of this study is the uncertainty regarding whether its findings, derived from grating scales, can be generalized to other types of textures. Future research should extend this investigation to a broader range of textures. However, unlike grating scales, general textures cannot be characterized by a single parameter, presenting a methodological challenge that needs to be addressed.

## V. CONCLUSIONS

In this study, we determined the error tolerances for visual and tactile grating stimuli presented via electrostatic friction displays, building on earlier studies [16], [17]. Thus far, no researchers investigated this for periodic and random grating scales by using surface texture displays. These results provide valuable insights for designers of haptic content and

developers of surface texture displays, enabling them to better understand the required consistency between visual and tactile stimuli and to define the necessary specifications for their applications and displays.

Future research should investigate wavelengths both larger and smaller than those examined in this study to establish comprehensive design guidelines for haptic content.

## REFERENCES

- [1] K. Otake, S. Okamoto, Y. Akiyama, and Y. Yamada, "Tactile texture rendering for electrostatic friction displays: Incorporation of low-frequency friction model and high-frequency textural model," *IEEE Transactions on Haptics*, vol. 15, no. 1, pp. 68–73, 2022.
- [2] C. Basdogan, F. Giraud, V. Levesque, and S. Choi, "A review of surface haptics: Enabling tactile effects on touch surfaces," *IEEE Transactions on Haptics*, vol. 13, no. 3, pp. 450–470, 2020.
- [3] R. V. Grigori and J. E. Colgate, "Closed loop application of electroadhesion for increased precision in texture rendering," *IEEE Transactions on Haptics*, vol. 13, no. 1, pp. 253–258, 2020.
- [4] T. Nakamura and A. Yamamoto, "A multi-user surface visuo-haptic display using electrostatic friction modulation and capacitive-type position sensing," *IEEE Transactions on Haptics*, vol. 9, no. 3, pp. 311–322, 2016.
- [5] S. E. Emgin, A. Aghakhani, T. M. Sezgin, and C. Basdogan, "Haptable: An interactive tabletop providing online haptic feedback for touch gestures," *IEEE Transactions on Visualization and Computer Graphics*, vol. 25, no. 9, pp. 2749–2762, 2019.
- [6] H. Shin, J.-M. Lim, J.-U. Lee, G. Lee, and K.-U. Kyung, "Effect of tactile feedback for button gui on mobile touch devices," *ETRI Journal*, vol. 36, no. 6, pp. 979–987, 2014.
- [7] F. Giraud, M. Amberg, and B. Lemaire-Semail, "Design and control of a haptic knob," *Sensors and Actuators A: Physical*, vol. 196, pp. 78–85, 2013.
- [8] H.-Y. Chen, J. Park, S. Dai, and H. Z. Tan, "Design and evaluation of identifiable key-click signals for mobile devices," *IEEE Transactions on Haptics*, vol. 4, no. 4, pp. 229–241, 2011.
- [9] E. Koskinen, T. Kaaresoja, and P. Laitinen, "Feel-good touch: Finding the most pleasant tactile feedback for a mobile touch screen button," in *Proceedings of the 10th international conference on Multimodal interfaces*, ser. ICMI '08. ACM, 2008, pp. 297–304.
- [10] V. Lévesque, L. Oram, K. MacLean, A. Cockburn, N. D. Marchuk, D. Johnson, J. E. Colgate, and M. A. Peshkin, "Enhancing physicality in touch interaction with programmable friction," in *Proceedings of ACM SIGCHI Conference on Human Factors in Computing Systems*, 2011, pp. 2481–2490.
- [11] S. Brewster, F. Chohan, and L. Brown, "Tactile feedback for mobile interactions," *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, pp. 159–162, 2007.
- [12] M. Fukumoto and T. Sugimura, "Active click: tactile feedback for touch panels," in *Proceedings of ACM SIGCHI 2001 the Conference on Human Factors in Computing Systems*, 2001, pp. 121–122.
- [13] G. Liu, C. Zhang, and X. Sun, "Tri-modal tactile display and its application into tactile perception of visualized surfaces," *IEEE Transactions on Haptics*, vol. 13, no. 4, pp. 733–744, 2020.
- [14] S.-C. Kim, A. Israr, and I. Poupyrev, "Tactile rendering of 3D features on touch surfaces," in *Proceedings of ACM Symposium on User Interface Software and Technology*, 2013, pp. 531–538.
- [15] J. R. Kim and H. Z. Tan, "A study of touch typing performance with keyclick feedback," in *2014 IEEE Haptics Symposium*, 2014, pp. 227–233.
- [16] S. Yamaguchi, S. Kaneko, and H. Kajimoto, "Measurement of the permissible range of consistency between visual and tactile presentations of line grating textures," *Applied Sciences*, vol. 10, no. 7, p. 2494, 2020.
- [17] A. C. Ai Kurita, Mirai Azechi and S. Okamoto, "Minimally allowable inconsistency between visual and synthetic tactile grating textures for surface tactile displays," in *Proceedings of IEEE Global Conference on Consumer Electronics*, 2024, pp. 761–763.
- [18] J. Scheibert, S. Leurent, A. Prevost, and G. Debrégeas, "The role of fingerprints in the coding of tactile information probed with a biomimetic sensor," *Science*, vol. 323, no. 5920, pp. 1503–1506, 2009.
- [19] A. İşleyen, Y. Vardar, and C. Basdogan, "Tactile roughness perception of virtual gratings by electrovibration," *IEEE Transactions on Haptics*, vol. 13, no. 3, pp. 562–570, 2020.
- [20] A. M. Smith, C. E. Chapman, M. Deslandes, J. S. Langlais, and M. P. Thibodeau, "Role of friction and tangential force variation in the subjective scaling of tactile roughness," *Experimental Brain Research*, vol. 144, no. 2, pp. 211–223, 2002.
- [21] M. M. Taylor and S. J. Lederman, "Tactile roughness of grooved surfaces: A model and the effect of friction," *Perception & Psychophysics*, vol. 17, no. 1, pp. 23–36, 1975.
- [22] S. Bocheureau, S. Sinclair, and V. Hayward, "Perceptual constancy in the reproduction of virtual tactile textures with surface displays," *ACM Transactions on Applied Perception*, vol. 15, no. 2, pp. 1–12, 2018.
- [23] L. A. Jones and N. B. Sarter, "Tactile displays: Guidance for their design and application," *Human Factors: The Journal of the Human Factors and Ergonomics Society*, vol. 50, no. 1, pp. 90–111, 2008.
- [24] G. D. Goff, "Differential discrimination of frequency of cutaneous mechanical vibration," *Journal of Experimental Psychology*, vol. 74, no. 2, pp. 294–299, 1967.
- [25] M. Rothenberg, R. T. Verrillo, S. A. Zahorian, M. L. Brachman, and S. J. Bolanowski, "Vibrotactile frequency for encoding a speech parameter," *Journal of Acoustical Society of America*, vol. 62, no. 4, pp. 1003–1012, 1977.
- [26] M. Azechi and S. Okamoto, "Combined virtual bumps and textures on electrostatic friction tactile displays," in *IEEE 11th Global Conference on Consumer Electronics*, 2022, pp. 315–317.
- [27] A. Chihara, M. Azechi, A. Kurita, and S. Okamoto, "Soft feel presentation on touch panels using low-frequency friction stimuli," in *IEEE 13th Global Conference on Consumer Electronics*, 2024.
- [28] T. Nakamura and A. Yamamoto, "Multi-finger electrostatic passive haptic feedback on a visual display," *Proceedings of IEEE World Haptics Conference*, pp. 37–42, 2013.
- [29] C. C. Wier, W. Jesteadt, and D. M. Green, "A comparison of method-of-adjustment and forced-choice procedures in frequency discrimination," *Perception & Psychophysics*, vol. 19, pp. 75–79, 1976.
- [30] W. Chen, Y. Hu, N. Ladevèze, and P. Bourdot, "Quick estimation of detection thresholds for redirected walking with method of adjustment," in *IEEE Conference on Virtual Reality and 3D User Interfaces*, 2019, pp. 878–879.
- [31] S. Kuroki, M. Sawayama, and S. Nishida, "The roles of lower- and higher-order surface statistics in tactile texture perception," *Journal of Neurophysiology*, vol. 126, no. 1, pp. 95–111, 2021.
- [32] H. Hasegawa, S. Okamoto, and Y. Yamada, "Phase difference between normal and shear forces during tactile exploration represents textural features," *IEEE Transactions on Haptics*, vol. 13, no. 1, pp. 11–17, 2020.
- [33] S. J. Bensmaïa and M. Hollins, "Complex tactile waveform discrimination," *Journal of the Acoustical Society of America*, vol. 108, pp. 1236–1245, 2000.
- [34] K. Otake, S. Okamoto, Y. Akiyama, and Y. Yamada, "Tactile texture display combining vibrotactile and electrostatic-friction stimuli: Substantial effects on realism and moderate effects on behavioral responses," *ACM Transactions on Applied Perception*, vol. 19, no. 4, p. 18, 2022.
- [35] J. R. Phillips and K. O. Johnson, "Tactile spatial resolution. II. Neural representation of bars, edges, and gratings in monkey primary afferents," *Journal of Neurophysiology*, vol. 46, no. 6, pp. 1192–1203, 1981.
- [36] C. Kaernbach, "Simple adaptive testing with the weighted up-down method," *Perception & Psychophysics*, vol. 49, no. 3, pp. 227–229, 1991.
- [37] M. R. Leek, "Adaptive procedures in psychophysical research," *Perception amp; Psychophysics*, vol. 63, no. 8, pp. 1279–1292, 2001.
- [38] C. Hutton, S. Ziccardi, J. Medina, and E. S. Rosenberg, "Individualized calibration of rotation gain thresholds for redirected walking," in *ICAT-EGVE 2018 - International Conference on Artificial Reality and Telexistence and Eurographics Symposium on Virtual Environments*. The Eurographics Association, 2018.
- [39] J. Grose-Fifer, *Sensation and perception*, C. Olman, Ed. CUNY Pressbooks, 2025.