

Effect of Continuous Transition between Force Stimuli on Haptic Force Discrimination Tasks

Ravi Prakash Meghwanshi

Department of Electrical Engineering
Indian Institute of Technology Jodhpur
Jodhpur, India
meghwanshi.1@iitj.ac.in

Amit Bhardwaj

Department of Electrical Engineering
Indian Institute of Technology Jodhpur
Jodhpur, India
amitb@iitj.ac.in

Himanshu Kumar

Department of Electrical Engineering
Indian Institute of Technology Jodhpur
Jodhpur, India
hkumar@iitj.ac.in

Abstract—Conventional haptic force discrimination tasks involve perceptual differentiation between stimulus pairs separated by a brief inter-stimulus interval (ISI). It is unclear whether the force JND measured using conventional methods, i.e., with ISI in-between stimuli, remains consistent in the case of continuous force discrimination. In this work, we investigate this consistency by studying the effect of transitions between force stimuli to understand continuous force discrimination. For this purpose, a psychophysical experiment is designed to examine two different transition conditions, viz. i) time-separated transition: force stimuli separated by an ISI, and ii) continuous transition: force stimuli linearly transitioned from one stimulus to another. We utilize the method of constant stimuli to study the effect of these transitions from the recorded haptic responses of the same 10 users for both conditions. We estimate the psychometric functions from the collected responses to determine the Weber fraction. The results show that the Weber fraction is significantly different and higher for the case of continuous transition compared to the conventional time-separated transition between force stimuli. These findings are significant for understanding the continuous haptic force perception and may be useful in many applications requiring continuous force perception, such as virtual reality, telepresence, and teleaction systems.

Index Terms—Force perception, Weber fraction, Just Noticeable Difference (JND), Inter-Stimulus Interval (ISI), psychophysics.

I. INTRODUCTION

Haptic perception is a fundamental sensory ability that enables us to engage with and understand the physical world through touch [1]. It allows us to identify, discriminate, and recognize objects, playing an essential role in daily tasks and interactions. This perception is driven by two key components: cutaneous [2], which involves the sense of touch through the skin, and kinesthetic [3], which relates to the awareness of body position and movement. In this work, we focus on kinesthetic force perception. Similar to other kinds of stimuli, human perceptual limitations for force stimuli are quantified by the *Just Noticeable Difference* (JND) or Weber fraction (δ) with respect to the reference force (F_r). The JND corresponds to the smallest perceivable change in F_r , whereas the Weber

fraction (δ) [4] is equal to the smallest perceivable relative change in F_r .

In standard classical psychophysical methods [5] for the JND computation, the reference (F_r) and comparison (F_c) force stimuli are kept constant and separated by a finite *Inter-Stimulus Interval* (ISI). However, this does not correspond to the practical applications where continuous force variation is encountered, and force discrimination is to be performed in the absence of ISI. For instance, in haptic robot teleoperation [6], users typically experience a continuous force rather than a discrete series of force stimuli. Further, continuous force perception is essential for advanced prosthetic limbs [7] to enable natural movement, as well as for virtual reality systems [8] that simulate realistic touch and interaction with virtual objects.

Previous studies on force perception assumed that the JND for the constant forces separated by the ISI generalizes the perception of the continuous forces, where the transition between force stimuli is continuous. In this study, we question this assumption and determine how the transition between force stimuli affects force perception. The impact of transition on the task of discriminating electrotactile frequencies is examined in a previous study [9]. For this, the authors examine three transition patterns between reference and test frequency: time-separated, step, and gradual. The comparison/test frequency is generated by employing the adaptive weighted staircase method [10]. A 2-alternative forced choice (2-AFC) paradigm [11] is utilized to determine the user's JND for stimulation frequency. The results indicate that the JND is lower in time-separated patterns compared to step and gradual patterns. This suggests that the sensitivity in discriminating frequencies significantly decreases when the transition between stimuli is gradual. This leads us to examine how the transition between force stimuli affects the force discrimination task.

In literature, numerous studies investigate force discrimination tasks, examining factors such as movement direction [12], speed [13], arm position [14], the effects of motion in virtual reality [15], and hand dominance [16]. Despite this extensive research into various facets of haptic force perception, there remains limited exploration of continuous force discrimination tasks, where continuous transition occurs between force stimuli. We also find some studies that explore the effect of

This work was supported in part by a grant from i-HUB Drishti, TIH Jodhpur for a project titled as "Haptics based Medical Simulator for Abdomen Palpation and Pulse Behavior" and in part by a grant from IHFC, TIH IIT Delhi for a project titled as "Telepresence and Teleaction System for Robot Assisted Dentistry". (Corresponding author: Amit Bhardwaj)

the inter-stimulus interval (ISI) on discrimination tasks for different stimuli [17]–[21]. The study [17] shows the impact of the inter-stimulus interval (ISI) on thermal perception. For this, the study examines the effect of three different ISIs, 0, 3, and 5 seconds, on thermal discrimination tasks. The authors utilize the 1-up/1-down adaptive staircase method to determine the thermal JND. The experiment also incorporates a reference temperature of 31°C , which is consistently lower than the standard and comparison thermal stimuli, providing a consistent perceptual baseline for comparing stimuli. The findings indicate that 3 seconds ISI results in a 12% increase in JND, while a 5 second ISI leads to a 21% increase in comparison to 0 seconds ISI. These results demonstrate a significant effect of ISI on thermal discrimination.

Similarly, the study [18] also examines the effect of ISI on the vibrotactile discrimination experiment. The study observed that users tend to make better discrimination judgments when the first stimulus is closer to the mean of the stimulus set compared to the second stimulus. This decision bias is influenced by a temporal phenomenon known as the time-order effect. The author examines how the varying inter-stimulus interval (ISI) and the distance from the first stimulus to the average of the stimulus set influence the time-order effect. The results reveal that the time-order effect varies non-linearly with the ISI, it is significantly more effective at short ISIs (300–600 *ms*).

Other studies [19] and [20] also show that the ISI between the stimuli impacts the ability to discriminate tone loudness for auditory stimuli and visual size for visual stimuli, respectively. Both findings indicate that JND increases with longer ISI, reflecting a diminished ability to differentiate between stimuli. Whereas the study [21] finds no effect of ISI on tasks such as facial emotion discrimination, which assesses users' ability to determine whether two consecutive novel faces are the same or different. The discussed studies collectively suggest that the transition between stimuli significantly influences haptic perception.

In the present study, we explore whether the JND/ Weber fraction estimated through the time-separated transition between force stimuli (with ISI) accurately predicts users' ability to differentiate changes in force stimuli when the transition is continuous (linearly varying). For this purpose, a psychophysical experiment is designed with two different transition conditions between the reference (F_r) and comparison (F_c) force stimuli. In the first condition, the force stimuli are separated by an inter-stimulus interval (ISI), while the transition is linear (from F_r to F_c or F_c to F_r) in the second condition. We employ a classical psychometric approach *Method of Constant Stimuli* [22] to determine the perceptual threshold, i.e., JND, and point of subjective equality (PSE). The estimated thresholds from both conditions are analyzed statistically to study the effect of the presence of ISI against continuous transition.

The main contributions of this work are:

- A study analyzing the effect of transition between the force stimuli on haptic perception of force discrimination

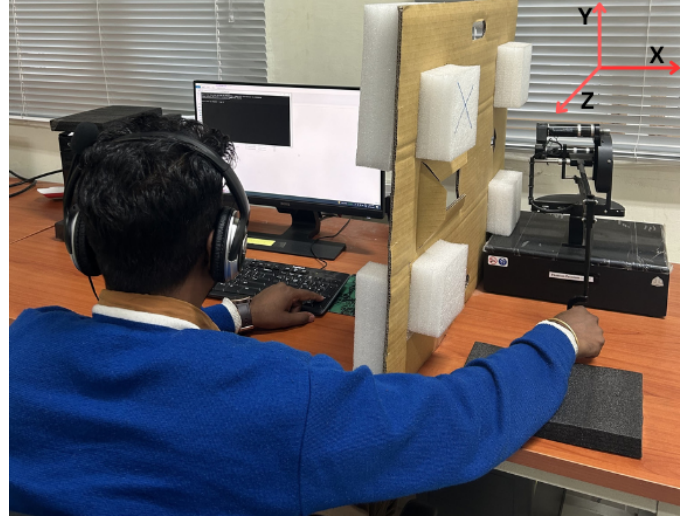


Fig. 1. Experimental Setup: a subject experiencing haptic force stimuli along the positive *Z*-axis via ‘Phantom Premium 1.5 HF’ device, masked from visual and auditory distractions.

in terms of the JND/ Weber fraction framework. The study utilizes the statistical framework to infer the significance of force stimuli transition period viz transition with ISI and continuous transition. We find that the transition between the force stimuli F_r and F_c do affect the haptic force perception in force discrimination.

- The results suggest that the ability to distinguish between time-separated force stimuli is consistently better than that of stimuli presented continuously, i.e., JND for force discrimination in the presence of ISI is much lower than JND for force discrimination in the case of continuous force.

The paper is organized as follows. Section II outlines the experimental setup and procedures used to estimate the perceptual thresholds for different transitions through two separate experimental conditions. Section III presents and compares the results obtained from both experiments. Section IV discusses the obtained result and the implications of that in the data-compression approach with future works. Finally, Section V concludes the paper.

II. METHODOLOGY

In this section, we describe our experimental setup and framework for data collection to study the force discrimination perception using two different transitions between the force stimuli.

A. Experimental Setup

We employ a force-feedback haptic interface, ‘PHANTOM Premium 1.5 HF’ (SensAble Technologies, Woburn, MA, USA), having six degrees of freedom along with an open source platform ‘CHAI3D’ [23], for our experimental setup. In the experiments, users are subjected to a one-dimensional (1D) haptic force stimulus along a positive *Z*-axis. The users are instructed to hold the stylus of the haptic device while keeping

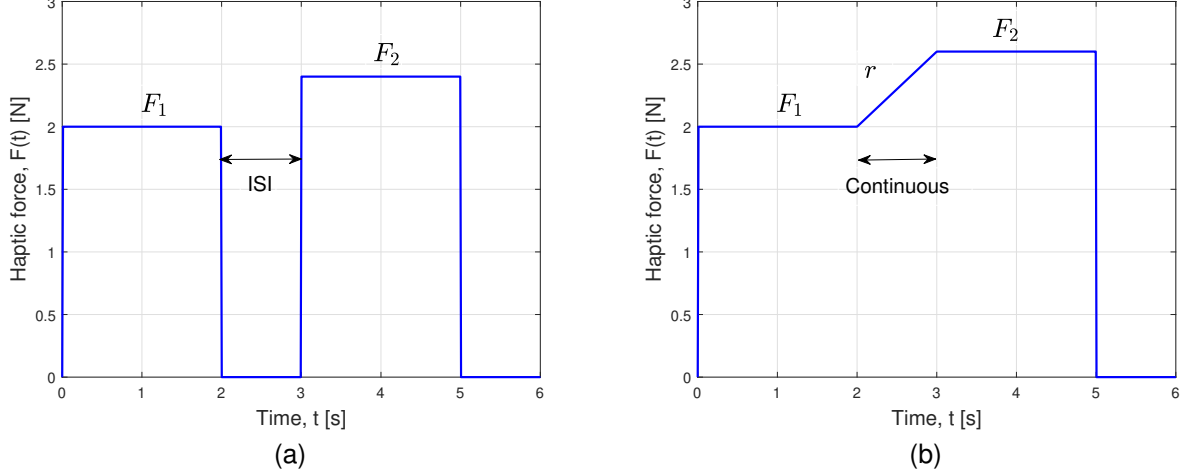


Fig. 2. Illustrations of the two transitions between force stimuli F_1 and F_2 used to estimate the force JND, which are: (a) Time-separated transition: separated by an inter-stimulus interval (ISI) for the first condition and, (b) Continuous linear transition: linearly varying for the second condition.

their arm fixed on the armrest during the force perception, as shown in Fig. 1. The users respond to the displayed question ‘which force is higher’ with the help of a ‘Keyboard’. The users press ‘1’ on the keyboard with their left hand when they perceive the first force stimuli higher than the second force stimuli and press ‘2’ otherwise. To minimize the external factors influencing haptic force perception, white noise is played through headphones to mask the audio cue. Along with this, cardboard is also placed between the users and the haptic device to block the visual cues. Figure 1 shows the experimental setup used for data collection along with the reference axis.

B. Force Stimuli

The study consists of a psychophysical experiment that provides two force stimuli, F_1 and F_2 , using two different transition patterns, as illustrated in Fig. 2. In the first condition, the force stimuli F_1 and F_2 are separated by an inter-stimulus interval (ISI) of 1 seconds, which is the conventional *time-separated transition*. In contrast, the second condition is designed to examine the effects of continuous transitions between force stimuli. For that, a linear transition with force-rate r between force stimuli F_1 and F_2 is applied over a duration of 1 second, which is defined as the *continuous linear transition*. The force-rate r is defined as the per unit time difference between forces F_1 and F_2 over a duration of 1 second, i.e., $r = (F_2 - F_1)$. Both haptic force stimuli, F_1 and F_2 , are maintained constant for 2 seconds in both conditions.

The profiles $F_I(t)$ and $F_{II}(t)$ of haptic force stimuli for the first and second conditions can be mathematically represented as functions of time t as in (1) and (2), respectively.

$$F_I(t) = \begin{cases} F_1 & \text{for } 0 \leq t \leq 2 \\ 0 & \text{for } 2 < t \leq 3 \\ F_2 & \text{for } 3 < t \leq 5 \end{cases} \quad (1)$$

$$F_{II}(t) = \begin{cases} F_1 & \text{for } 0 \leq t \leq 2 \\ F_1 + r(t - 2) & \text{for } 2 < t \leq 3 \\ F_2 & \text{for } 3 < t \leq 5. \end{cases} \quad (2)$$

To evaluate users’ ability to discriminate between forces using two different transitions (time-separated and linearly continuous), a psychophysical experiment is conducted. In this context, one of the force stimuli between F_1 and F_2 is defined as the reference force stimulus (F_r), while the other is defined as the comparison force stimulus (F_c) in a random manner. For this study, we choose the reference force stimulus (F_r) as a constant force with a magnitude of $2N$. The comparison force stimuli (F_c) are defined from set $F_{c1} = \{1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8\} N$, and $F_{c2} = \{1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0\} N$ for the first and second conditions, with nine and eleven force levels, respectively. The levels of comparison force stimuli (F_{c1} or F_{c2}) are selected such that the force stimulus with the highest magnitude is almost always perceived as greater than the reference force, and the force stimulus with the lowest magnitude is almost always perceived as less than the reference force, while maintaining a constant step size between the consecutive force stimuli. The force-rate (r) ranges from 0 to $1 N/s$ as per selected F_{c2} in the second condition. After every trial, the users answer the displayed question related to their haptic perception of the force stimuli.

C. Procedure

Users conduct the designed psychophysical experiment using the method of constant stimuli on the experimental setup shown in Fig. 1. In this study, we used the method of constant stimuli rather than an adaptive method [24] because it enables full estimation of the psychometric function (which includes PSE, JND, and slope of curve), ensures consistent

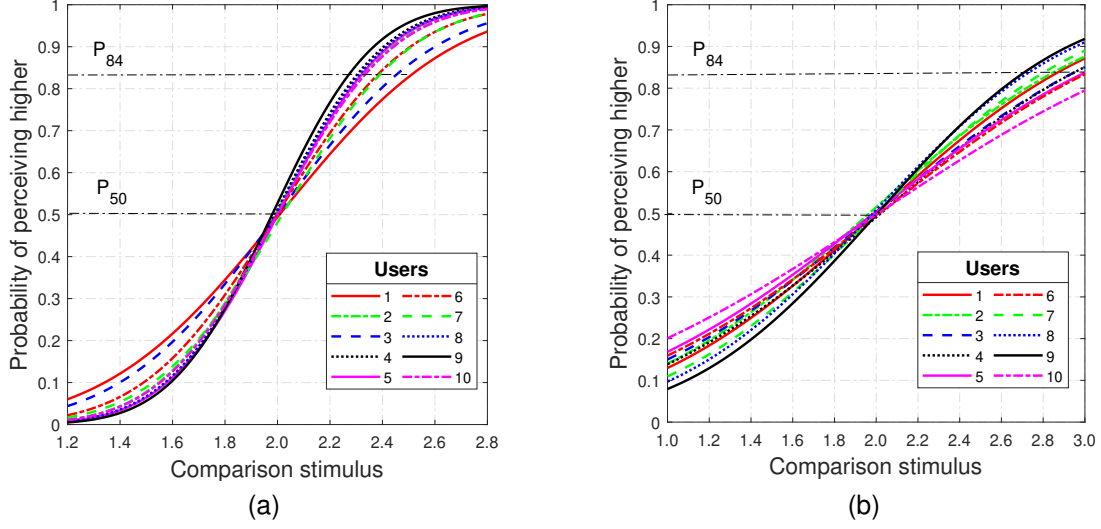


Fig. 3. Psychometric curves for force discrimination for $2N$ reference force (F_r) for all users, when transition period between force stimuli is (a) separated by an inter-stimulus interval (ISI) for the first condition and, (b) linearly varying for the second condition. The horizontal line at P_{50} indicates the point of subjective equality (PSE), and the line at P_{84} corresponds to the threshold criterion used to measure the JND.

stimulus presentation across users, and avoids potential biases introduced by early responses in adaptive procedures. Before starting the experiment, users receive a brief introduction and training on the haptic device to ensure they feel comfortable. In each trial, users perceive a reference force stimulus (F_r) with a constant magnitude throughout the experiment and a comparison force stimulus (F_c) that varies across trials. The presentation order of reference force stimulus (F_r) and comparison force stimulus (F_c) is randomized across trials to avoid the time-order error. The order of transition conditions is also randomized across users. The users are asked to indicate which of the two stimuli is higher in a 2-AFC paradigm. To minimize user response bias, each level of comparison force stimuli (F_c) is presented 50 times in a pseudo-randomized order. The same procedure is used for both transition conditions.

D. Data Collection

We collected the responses of the same ten users (8 males and 2 females, aged 20 to 30 years, all right-handed) for both designed experimental conditions discussed above, and none of them suffers from any neuro-physiological disorder. Before the start of the experiment, written consent was taken from all the users. The experimental procedures used for data collection are approved by the Institutional Ethical Committee at IIT Jodhpur (IEC/IITJ/2023-24/04).

Each user records a total of 450 ($9 F_c \times 50$ trials) and 550 ($11 F_c \times 50$ trials) trials for the first and second conditions, respectively. Each trial lasts approximately 5 to 8 seconds, including user response time. This results in the total duration of the experiment being approximately 2.5 hours per user. To prevent perceptual fatigue due to long duration in a single sitting, the experiment is divided into multiple sessions, with each session lasting no more than 20 minutes per day for a

user. Along with this, the users are allowed to take breaks at any time during the session.

III. RESULTS

We analyze the responses of each user for both conditions with the help of a psychometric function. For this, the psychometric function is estimated by fitting Cumulative Gaussian Functions to the responses gathered during each transition condition [25]. Figure 3 shows the psychometric curve for both conditions across all the users. The estimated psychometric functions are characterized in terms of JND, the point of subjective equality (PSE), and the slope. The PSE refers to the comparison stimulus level at which the two stimuli appear to be the same, and thus, it refers to the 50 percentile point P_{50} in the psychometric function, i.e., $PSE = P_{50}$ as shown in Fig. 3. The force JND is computed by taking the stimulus difference between the stimulus level at proportion 0.84 and proportion 0.50, i.e., $JND = P_{84} - P_{50} = \sigma$, where σ is the standard deviation. The relative force JND or Weber fraction (δ) is defined as the JND normalized by the reference force $F_r = 2N$ and computed as in (3).

$$\text{Weber fraction } (\delta) = \frac{P_{84} - P_{50}}{F_r} \quad (3)$$

The Constant Error (CE), computed as given in (4), refers to the systematic bias in a user's responses, defined by the deviation of the PSE from the actual reference force stimulus.

$$\text{Constant Error (CE)} = PSE - F_r \quad (4)$$

The slope of the psychometric function, estimated at P_{50} according to [25], reflects perceptual sensitivity. For a given PSE, a steeper slope (small σ) corresponds to a lower Weber fraction, which reflects improved relative discriminability

TABLE I
PSYCHOMETRIC MEASURES (WF, PSE, CE, SLOPE) FOR BOTH TRANSITION CONDITIONS ACROSS ALL USERS.

Users	Transition conditions							
	Time-separated transition				Continuous linear transition			
	Weber fraction (δ)	PSE	Constant Error (CE)	Slope	Weber fraction (δ)	PSE	Constant Error (CE)	Slope
1	0.2587	2.0072	0.0072	0.7669	0.4404	1.9970	-0.0030	0.4509
2	0.1730	1.9912	-0.0088	1.1506	0.4444	1.9670	-0.0330	0.4457
3	0.2330	1.9992	-0.0008	0.8508	0.4795	1.9990	-0.0010	0.4138
4	0.1610	1.9896	-0.0104	1.2316	0.4685	2.0210	0.0210	0.4240
5	0.1642	2.0008	0.0008	1.2099	0.5055	1.9890	-0.0110	0.3877
6	0.1978	1.9976	-0.0024	1.0048	0.4925	2.0150	0.0150	0.3921
7	0.1906	2.0184	0.0184	1.0376	0.4054	2.0010	0.0010	0.4898
8	0.1634	1.9944	-0.0056	1.2097	0.3764	1.9850	-0.0150	0.5263
9	0.1497	1.9800	-0.0200	1.3204	0.3544	2.0070	0.0070	0.5590
10	0.1730	1.9976	-0.0024	1.1483	0.4955	2.0090	0.0090	0.3313
Mean	0.1864	1.9976	-0.0024	1.0931	0.4463	1.9990	-0.0010	0.4421

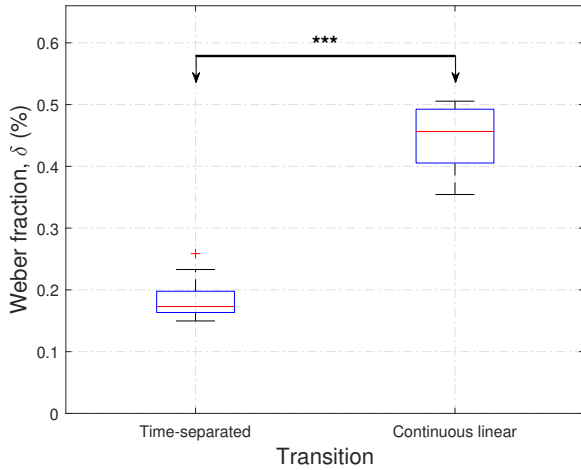


Fig. 4. Box-plot analysis of the Weber fraction (δ) for both transition conditions (time-separated and continuous linear) for the reference force $F_r = 2N$ along with statistical significance represented by *** ($p = 0.000$).

(higher sensitivity) and confirms the inverse relationship between slope and Weber fraction. Table I includes the Weber fractions, PSEs, constant errors (CEs), and slopes for both transition conditions across all users.

For the first condition, which utilizes a time-separated transition, the Weber fraction lies in the range of [14.97, 19.78]% for all users, except for users 3 and 5. For users 3 and 5, the values of the Weber fraction are 23.30% and 25.87%, respectively. The values of PSE, Constant Error (CE), and slope lie in the range of [1.9800, 2.0184], [-0.0200, 0.0184] and [0.7669, 1.3204] for all users, respectively. On average, the Weber fraction, PSE, CE, and slope across all users are 18.64%, 1.9976, -0.0024, and 1.0931, respectively.

Similarly, the Weber fraction lies in the range of [44.04, 50.55]% for all users, except for users 7, 8, and 9 for the second condition, which employed a continuous linear transition. For users 7, 8 and 9, the values of Weber fraction are 40.54%, 37.64% and 35.44%, respectively. The values of PSE, Constant Error (CE), and slope lie in the range of

[1.9670, 2.0210], [-0.0330, 0.0210] and [0.3313, 0.5590] for all users, respectively. On average, the Weber fraction, PSE, CE, and slope across all users are 44.63%, 1.9990, -0.0010, and 0.4421, respectively.

Further, we determine whether the observed values of Weber fraction, PSE, Constant Error (CE), and slope are statistically different between the time-separated and continuous linear transition. To do this, we perform a paired sample t-test to compare the corresponding values. Prior to the analysis, the normality of the data is tested using the Shapiro-Wilk test. Figure 4 shows the box plot analysis of the Weber fraction (δ). The results of the analysis indicate a significant difference between the transition patterns regarding the Weber fraction, with $t(9) = -14.80, p = 0.000$ and for slope, with $t(9) = 11.94, p = 0.000$. Whereas, no significant difference is observed regarding the PSE and Constant Error (CE), with $t(9) = -0.23, p = 0.824$.

IV. DISCUSSION

Our results show that the Weber fraction is higher when the transition between the force stimuli is linearly varied compared to the conventional method. The higher values of the Weber fraction suggest that it is harder to discriminate force stimuli with linearly continuous change. The Weber fraction for the time-separated transition aligns closely with established findings in force perception, as various studies have reported similar Weber fraction values [26]–[30].

It is also observed that a steeper slope in the continuous transition condition indicates that the user is more sensitive compared to the condition with the inter-stimulus interval (ISI). Also, the near-zero Constant Error (CE) indicates both user accuracy and experimental validity, showing reliable and unbiased performance for both conditions.

We also interpret our results on continuous force discrimination in the context of haptic data compression. In the literature, a perceptually adaptive sampling approach based on Weber's law of perception has been employed for haptic data compression for a typical teleoperation application [30]–[32]. This approach transmits only perceptually relevant data points. Points that fall outside the perceptual deadband are considered

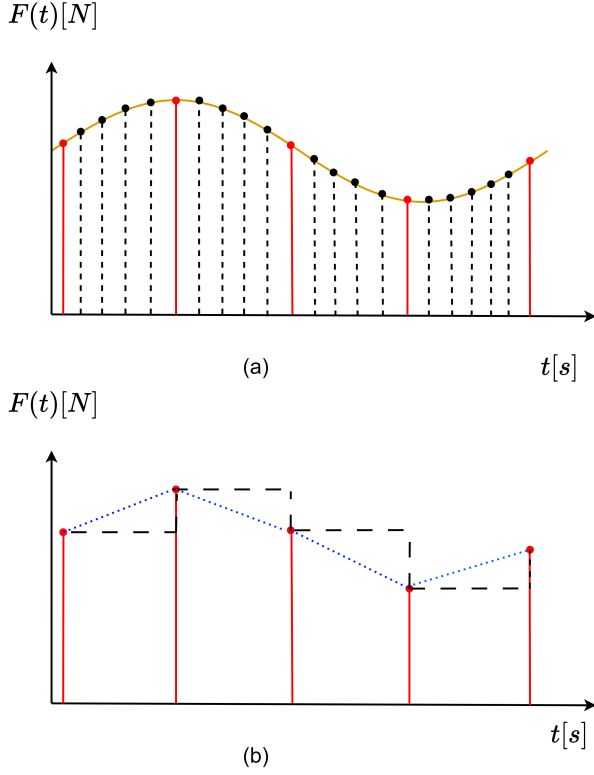


Fig. 5. (a) An illustration of 1D continuous force stimuli $F(t)$ vs time (t) at the transmitter side (brown color), where red data points are perceptually transmitted using Weber's law, (b) reconstruction of the signal using sample-and-hold (black-dashed line) and linear interpolation (blue-dashed line) at the receiver side.

perceptually relevant samples. At the receiver end, the signal is reconstructed using a standard interpolation method, such as sample and hold, as shown in Fig. 5 (black-dashed line). In the received signal, there is a sharp transition between any two consecutive points, which is the case of zero ISI.

To incorporate the perception of continuous force, one should allow force to vary gradually from one point to another at the receiver end. This will lead to further data reduction since the Weber fraction for the continuous force perception case is higher than the case when force stimuli are separated by a fixed ISI. However, to incorporate continuous force perception into perceptual haptic data compression, the interpolation method must be modified, for example, by using linear interpolation (blue-dashed line). Implementing this will require transmitting the slope of the signal (i.e., the rate of change) alongside the perceptually relevant samples. This will be validated in future experiments.

The next step in this line of research is to compare the results of the continuous linear transition case (the present study) with those when the ISI equals zero. Based on studies in the other domains [17], we expect that the JND will decrease with a reduction in the ISI. We will investigate this issue in future work. Additionally, the current work has been conducted with only one reference force. It would be interesting to explore

how the JND for continuous force perception varies when using different reference stimuli. Will it adhere to Weber's law of perception, similar to constant force discrimination? Furthermore, future studies could consider incorporating user-initiated movement within the current experimental framework. This would allow us to examine how active engagement affects continuous force perception and whether the findings are applicable to more interactive, real-world situations.

This study uncovers an important aspect of continuous haptic force discrimination which is crucial for enhancing continuous haptic force-feedback control systems [33], which provide a sense of touch through technology, such as in virtual reality or robotic controls. This study will help in making these systems more accurate and optimal for touch-based interactions. Additionally, this study holds a broader significance for fundamental research about human perception of the continuous haptic force.

V. CONCLUSION

In this study, we carried out a psychophysical experiment to examine how transitioning between two force stimuli affects force discrimination. We conducted an experiment with two transition conditions using the method of constant stimuli, where reference and comparison force stimuli were presented both in a conventional (separated by an ISI) and continuous linear manner. Both transition conditions were performed by the same 10 users. The results conclude that the Weber fraction is higher when the transition between force stimuli is continuous compared to the conventional method. This understanding of dynamics is important for applications in haptic technology, rehabilitation, and human-computer interaction, where continuous force feedback is essential. In the future, we would like to extend this work to study the effect of different continuous transition patterns between the force stimuli in haptic force discrimination.

REFERENCES

- [1] F. E. Van Beek, W. M. Bergmann Tiest, W. Mugge, and A. M. Kappers, "Haptic perception of force magnitude and its relation to postural arm dynamics in 3d," *Scientific reports*, vol. 5, no. 1, p. 18004, 2015.
- [2] D. Prattichizzo, C. Pacchierotti, and G. Rosati, "Cutaneous force feedback as a sensory subtraction technique in haptics," *IEEE Transactions on Haptics*, vol. 5, no. 4, pp. 289–300, 2012.
- [3] S. Chaudhuri and A. Bhardwaj, "Kinesthetic perception," *A Machine Learning Approach*, 2018.
- [4] E. H. Weber, *EH Weber on the tactile senses*. Psychology Press, 1996.
- [5] L. A. Jones and H. Z. Tan, "Application of psychophysical techniques to haptic research," *IEEE transactions on haptics*, vol. 6, no. 3, pp. 268–284, 2012.
- [6] R. V. Patel, S. F. Atashzar, and M. Tavakoli, "Haptic feedback and force-based teleoperation in surgical robotics," *Proceedings of the IEEE*, vol. 110, no. 7, pp. 1012–1027, 2022.
- [7] R. E. Fan, M. O. Culjat, C.-H. King, M. L. Franco, R. Boryk, J. W. Bisley, E. Dutson, and W. S. Grundfest, "A haptic feedback system for lower-limb prostheses," *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 16, no. 3, pp. 270–277, 2008.
- [8] Y. Shi and G. Shen, "Haptic sensing and feedback techniques toward virtual reality," *Research*, vol. 7, p. 0333, 2024.
- [9] S. Gholinezhad, S. Dosen, and J. Dideriksen, "Continuous transition impairs discrimination of electrotactile frequencies," *IEEE Transactions on Haptics*, vol. 15, no. 4, pp. 753–758, 2022.

- [10] C. Kaernbach *et al.*, “Simple adaptive testing with the weighted up-down method,” *Perception & psychophysics*, vol. 49, no. 3, pp. 227–229, 1991.
- [11] M. J. Hautus, N. A. Macmillan, and C. D. Creelman, *Detection theory: A user’s guide*. Routledge, 2021.
- [12] E. Dorjgotov, G. R. Bertoline, L. Arns, Z. Pizlo, and S. R. Dunlop, “Force amplitude perception in six orthogonal directions,” in *2008 Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems*. IEEE, 2008, pp. 121–127.
- [13] X.-D. Yang, W. F. Bischof, and P. Boulanger, “Perception of haptic force magnitude during hand movements,” in *2008 IEEE International Conference on Robotics and Automation*. IEEE, 2008, pp. 2061–2066.
- [14] S. Feyzabadi, S. Straube, M. Folgheraiter, E. A. Kirchner, S. K. Kim, and J. C. Albiez, “Human force discrimination during active arm motion for force feedback design,” *IEEE transactions on haptics*, vol. 6, no. 3, pp. 309–319, 2013.
- [15] T. Long, R. D. Giacalone, and A. T. Asbeck, “Directional perception of force in a virtual reality environment,” *IEEE Access*, vol. 9, pp. 110 346–110 358, 2021.
- [16] S. Kakade, S. Chaudhuri, and A. Gupta, “Bi-manual sensory discrimination: A kinesthetic study,” *IEEE Transactions on Haptics*, 2024.
- [17] M. Hojatmadani, B. Rigsby, and K. B. Reed, “Time delay affects thermal discrimination,” *IEEE Transactions on Haptics*, vol. 15, no. 2, pp. 451–457, 2022.
- [18] M. Karim, J. A. Harris, A. Langdon, and M. Breakspear, “The influence of prior experience and expected timing on vibrotactile discrimination,” *Frontiers in neuroscience*, vol. 7, p. 255, 2013.
- [19] Å. Hellström, “Comparison is not just subtraction: Effects of time- and space-order on subjective stimulus difference,” *Perception & psychophysics*, vol. 65, no. 7, pp. 1161–1177, 2003.
- [20] F. Gerr and R. Letz, “Covariates of human peripheral nerve function: Ii. vibrotactile and thermal thresholds,” *Neurotoxicology and teratology*, vol. 16, no. 1, pp. 105–112, 1994.
- [21] É. M. Bankó, V. Gál, and Z. Vidnyánszky, “Flawless visual short-term memory for facial emotional expressions,” *Journal of Vision*, vol. 9, no. 1, pp. 12–12, 2009.
- [22] W. A. Simpson, “The method of constant stimuli is efficient,” *Perception & psychophysics*, vol. 44, pp. 433–436, 1988.
- [23] F. Conti, F. Barbagli, R. Balaniuk, M. Halg, C. Lu, D. Morris, L. Sentis, J. Warren, O. Khatib, and K. Salisbury, “The chai libraries,” in *Proceedings of Eurohaptics 2003*, Dublin, Ireland, 2003, pp. 496–500.
- [24] M. R. Leek, “Adaptive procedures in psychophysical research,” *Perception & psychophysics*, vol. 63, no. 8, pp. 1279–1292, 2001.
- [25] K. Zchaluk and D. H. Foster, “Model-free estimation of the psychometric function,” *Attention, Perception, & Psychophysics*, vol. 71, no. 6, pp. 1414–1425, 2009.
- [26] X.-D. Pang, H. Z. Tan, and N. I. Durlach, “Manual discrimination of force using active finger motion,” *Perception & psychophysics*, vol. 49, no. 6, pp. 531–540, 1991.
- [27] H. Z. Tan, M. A. Srinivasan, B. Eberman, and B. Cheng, “Human factors for the design of force-reflecting haptic interfaces,” *Dynamic Systems and Control*, vol. 55, no. 1, pp. 353–359, 1994.
- [28] S. Allin, Y. Matsuoka, and R. Klatzky, “Measuring just noticeable differences for haptic force feedback: implications for rehabilitation,” in *Proceedings 10th Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems. HAPTICS 2002*. IEEE, 2002, pp. 299–302.
- [29] L. A. Jones and I. W. Hunter, “A perceptual analysis of stiffness,” *Experimental Brain Research*, vol. 79, pp. 150–156, 1990.
- [30] E. Steinbach, M. Strese, M. Eid, X. Liu, A. Bhardwaj, Q. Liu, M. Al-Ja’afreh, T. Mahmoodi, R. Hassen, A. El Saddik *et al.*, “Haptic codecs for the tactile internet,” *Proceedings of the IEEE*, vol. 107, no. 2, pp. 447–470, 2018.
- [31] R. Hinterseer, E. Steinbach, and S. Chaudhuri, “Perception-based compression of haptic data streams using kalman filters,” in *Acoustics, Speech and Signal Processing, 2006. ICASSP 2006 Proceedings. 2006 IEEE International Conference on*, vol. 5, may 2006, p. V.
- [32] A. Bhardwaj, B. Cizmeci, E. Steinbach, Q. Liu, M. Eid, J. AraUjo, A. El Saddik, R. Kundu, X. Liu, O. Holland *et al.*, “A candidate hardware and software reference setup for kinesthetic codec standardization,” in *2017 IEEE international symposium on haptic, audio and visual environments and games (HAVE)*. IEEE, 2017, pp. 1–6.
- [33] C. R. Carignan and K. R. Cleary, “Closed-loop force control for haptic simulation of virtual environments,” *Haptics-e*, vol. 1, no. 2, pp. 1–14, 2000.