Analysis of Static Eccentricity on the Position Estimation of Zero-Sequence Voltage Based Sensorless Techniques

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Abstract—The change of phase inductance due to faults, such as eccentricity, in electrical machines has been studied extensively in literature. These changes also affect the measurements of anisotropy based sensorless techniques, such as the zero-sequence voltage based techniques, by introducing an error in the estimated rotor position. This work presents an analysis of this error by means of an analytical solution of the inductances coupled with a sensitivity analysis of the applied zero-sequence voltage based sensorless technique DFC with a focus on PMSMs. The analytically calculated error in the position estimation is then verified with experimental results.

Index Terms—Eccentricity, PMSM, Sensorless Techniques, Zero-sequence Voltage

I. INTRODUCTION

The introduction of high efficiency electrical machines, such as the PMSM (Permanent Magnet Synchronous Machine), has led to an increasing demand to reduce both the complexity and cost of such machines. One of the aspects thoroughly investigated in this scope is the acquisition of rotor position information which is required for driving such machines. Instead of using mechanical sensors like encoders or resolvers, the group of so-called sensorless techniques can be applied either as a standalone solution or a redundancy.

The application of sensorless techniques for estimating the position of the rotor has been investigated thoroughly in literature [1]. They range from Back-EMF (Electromotive Force) based techniques, as presented in [2], to anisotropy based techniques which can be further grouped into current injection based techniques as presented in [3] and [4] and zero-sequence voltage based ones like [5]. As the name suggests, the measurements performed using these anisotropy based techniques rely on rotor position dependent parameters such as the phase inductances.

Previous research has shown that these inductances are affected by faults such as winding insulation faults [6] and eccentricity [7]. In fact, eccentricity is a fault which is commonly introduced in small amounts due to, for example, manufacturing tolerances [8]. By using inductance models, such as the MWFA (Modified Winding Function Approach), it is possible to predict the change in inductance for a specific characteristic of eccentricity. Although this aspect has been investigated for injection based techniques, as presented in [9], this change in inductance is usually not included in the mathematical description of the phase inductances used for zero-sequence voltage based sensorless techniques. Thus, these faults may introduce an unexpected deviation in the measured voltages which subsequently introduce an error in the estimation of the rotor position. Depending on the magnitude of the error and the required accuracy of the rotor position, this effect has to be considered in order to guarantee driving the electrical machine in a safe manner.

The specific zero-sequence voltage based technique investigated and applied in the presented research is called DFC (Direct Flux Control). This sensorless technique was first introduced by Strothmann in [10] and further developed for improved measurements [11] and in combination with different methods to cover a wide range of speed [12].

The goal of the presented research is to assess this error in position estimation due to SE (static eccentricity). For this, a model of the phase inductances of a PMSM affected by SE based on the MWFA is introduced in Section II. Afterwards, the zero-sequence voltage based sensorless technique DFC is recapped and a sensitivity analysis of the zero-sequence voltage measurements and the estimated electrical angle due to changes in the inductance matrix is presented in Section III. Using this mathematical description, analytical results of the position estimation and the error introduced by SE are presented. Finally, these results are compared with experimental results in Section IV.

II. MODEL OF PHASE INDUCTANCES AFFECTED BY STATIC ECCENTRICITY

The MWFA is based on the Winding Function Theory with the extended capability of including a non-symmetric air-gap between rotor and stator. This makes it a capable tool for modeling the inductances of eccentricity affected machines, as presented in [13], although the assumption of a linear soft magnetic material with an infinite magnetic permeability has to be considered for highly eccentric machines with a minimum air-gap close to zero. In fact, the

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calculated phase inductances of such machines, although not safe to drive, will show a continuously rising behavior with decreasing air-gap length instead of saturating.

Using the MWFA as proposed in [14], the rotor position \( \theta \) dependent inductance \( L_{xy} (\theta) \) is calculated using

\[
L_{xy} (\theta) = \mu_0 r l \int_0^{2\pi} N_y (\varphi, \theta) n_x (\varphi) g^{-1} (\varphi, \theta) \, d\varphi, \tag{1}
\]

with \( \mu_0 \) being the magnetic permeability of free space, \( r \) the mean radius of the air-gap and \( l \) the stack length. The parameter \( \varphi \) is an angle in the stator reference frame used for integration purposes, \( n_x (\varphi) \) the turns function of phase \( x \in \{a, b, c\} \) and \( N_y (\varphi, \theta) \) the modified winding function of phase \( y \in \{a, b, c\} \). This latter function also depends on the inverse air-gap function \( g^{-1} (\varphi, \theta) \), which is the function depending on the magnitude \( \delta_s \in [0, 1] \) and angular position \( \beta_s \) of the SE as portrayed in Figure 1 in the stator reference frame. Both these values depend on the relative position of rotor symmetry center \( O_r \) and stator symmetry center \( O_s \) as well as the mean air-gap \( g_0 \) of the non-eccentric machine such that

\[
\delta_s = \frac{|O_s - O_r|}{g_0}. \tag{2}
\]

Thus, this value represents a fraction of the air-gap and indicates the minimum air-gap length. Unlike the other two cases of eccentricity, i.e. dynamic and mixed eccentricity, the rotation center \( O_s \) is aligned with \( O_r \). Therefore, the position of the minimum air-gap \( \beta_s \) is constant in the stator reference frame.

The representation of the turns and air-gap functions is generic in equation (1). However, using Fourier-series with an arbitrary number of harmonics for both \( n_x (\varphi) \) and \( g^{-1} (\varphi, \theta) \) has led to a closed solution of equation (1) presented in [15]. The proposed solution covers all types of eccentricity but can be simplified if only SE is present in the machine. This simplification leads to an equation of type

\[
L_{xy} (\theta) = L_0 + \sum_{k=1}^{N} L_k \cos (k\theta - \alpha_k), \tag{3}
\]

where \( L_{xy} (\theta) \) is the inductance of phases \( x \) and \( y \), \( L_0 \) the mean inductance, \( L_k \) the magnitude of the \( k^{th} \) harmonic and \( \alpha_k \) its respective phase shift. If \( x = y \), equation (3) represents a self inductance, otherwise a mutual inductance.

By inserting values for \( \delta_s \) and \( \beta_s \) into equation (3), the change of the self inductance \( L_{aa} \) can be visualized as portrayed in Figure 2 for a PMSM with 12 slots and 10 poles. For simplicity, \( \beta_s = 45^\circ \) for all presented calculations. Figure 3 instead, shows the harmonic content of this inductance obtained by Fourier analysis, clearly indicating a significant change in the mean value. Although there is a change in the existing harmonics, it is considerably smaller than the one in the mean value. Furthermore, no additional harmonics are introduced due to SE outside of what can be attributed to numerical imprecision. These observations could be used to simplify this model, for example in order to monitor this variation on an embedded system, but in this analysis the complete expression will be used.

III. DFC SENSITIVITY ANALYSIS

The zero-sequence voltage based sensorless technique used in this investigation is DFC (Direct Flux Control). Although it requires the star-point of the machine to be accessible, it is characterized by good signal-to-noise ratio signals and it is computationally light. As shown in Figure 4 the DFC measurement vector in the phase
reference frame, $\Gamma_{abc}$, is assessed by measuring the voltage difference between a virtual star-point made of resistors and the real star-point before and after applying the PWM excitation per each phase [17]. This vector can be expressed as

$$\Gamma_{abc} = \begin{bmatrix} \Gamma_a \\ \Gamma_b \\ \Gamma_c \end{bmatrix} = \left( \sigma \left( L \left( \theta \right)^{-1} \right)^T - \frac{1}{3} I \right) T^T v_{DC},$$

(4)

with $L \left( \theta \right)$ being the matrix of the phase inductances, $I$ the identity matrix, $v_{DC}$ the bus voltage of the inverter, $T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and

$$\sigma = \frac{1}{T \left( L \left( \theta \right)^{-1} \right)^T T^T}.$$

(5)

By applying the Clarke Transformation Matrix $T_c$, the DFC measurement vector can be expressed in the stator reference frame as

$$\Gamma_{\alpha\beta\gamma} = \begin{bmatrix} \Gamma_\alpha \\ \Gamma_\beta \\ \Gamma_\gamma \end{bmatrix}^T.$$

(6)

The phase $\chi$ of this vector may be calculated by means of

$$\chi = \arctan \left( \frac{\Gamma_\beta}{\Gamma_\alpha} \right).$$

(7)

As shown in [18], this angle ranges between $-90^\circ$ and $90^\circ$ twice per each electrical period, making an estimation of the electrical angle possible but with an uncertainty of $180^\circ$. A number of ways to solve this duality have been proposed, such as the one in [19], but it will not be presented here for sake of brevity.

A sensitivity analysis proposed in [20] has shown analytically that the angle $\chi$ is susceptible to changes in the DFC measurements $\Gamma_{abc}$ and, thus, to changes in the inductances. This result has already been applied to estimate the change that a short circuit introduces to the zero-sequence voltage measurements as presented in [21].

Starting from a deviation matrix $\Delta L$ from the ideal inductance matrix $L^*$, such that the real inductance matrix is

$$L = L^* + \Delta L,$$

(8)

a deviation $\Delta \Gamma_{abc}$ from the ideal DFC measurement vector $\Gamma_{abc}^*$ is introduced by means of

$$\Delta \Gamma_{abc} = TL^* \left[ \Delta \sigma I - \sigma \Delta L^* L^{-1} \right],$$

(9)

$$\sigma = \frac{1}{T \left( L^* - \sigma \Delta L^* L^{-1} \right)^T T^T},$$

(10)

$$\Delta \sigma = \frac{TL^* \Delta LL^{-1} \sigma}{3T \left( L^* - \sigma \Delta L^* L^{-1} \right)^T T^T},$$

(11)

The derivation of these equations is omitted for reasons of brevity but may be found in [20].

Figure 5 illustrates $\Gamma_{abc}$ of both the symmetric machine and the one affected by SE with the harmonic content of $\Gamma_a$ portrayed in Figure 6. Similar to the phase inductance, a large change in the mean value is introduced by SE. However, the relevant part for estimating the position is the $2^{th}$ harmonic, i.e. the $10^{th}$ harmonic in this case, which also experiences a significant deviation. The remaining harmonics change to a lesser extent and no additional harmonics are introduced due to the applied SE.

Analog to $L$, one can write

$$\Gamma_{abc} = \Gamma_{abc}^* + \Delta \Gamma_{abc},$$

(12)

leading to the expression

$$\chi = \chi^* + \Delta \chi,$$

(13)

where the real angle $\chi$ consists of an ideal part $\chi^*$ and a deviation $\Delta \chi$. By defining

$$\Gamma_{\alpha\beta\gamma} = T_c \Gamma_{abc}$$

and

$$\Gamma_{\alpha\beta\gamma} = \Gamma_{\alpha\beta\gamma}^* + T_c \Delta \Gamma_{abc}$$

(14)

$$\Gamma_{\alpha\beta\gamma} = \Gamma_{\alpha\beta\gamma}^* + \Delta \Gamma_{\alpha\beta\gamma},$$

The mechanical angle (deg)
this deviation can be calculated as:

\[
\Delta \chi = \arctan 2 \left( \frac{\xi_\beta}{\xi_\alpha} \right),
\]

\[
\xi_\alpha = \frac{\Gamma_\alpha^2 + \Gamma_\alpha \Delta \Gamma_\alpha + \Gamma_\beta \Delta \Gamma_\beta + \Gamma_\beta^2}{\Gamma_\alpha^2 + \Gamma_\beta^2},
\]

\[
\xi_\beta = \frac{\Gamma_\alpha \Delta \Gamma_\beta - \Gamma_\beta \Delta \Gamma_\alpha}{\Gamma_\alpha^2 + \Gamma_\beta^2}.
\]

Using this mathematical framework along with equation (3), the error \(\Delta \chi\) due to a specific SE can be calculated. Results of \(\chi\) for a characteristic of SE and the resulting deviation \(\Delta \chi\) from \(\chi^*\), which is defined by \(\delta_s = 0.0\), are presented in Figure 7 and Figure 8 respectively. The resulting error has an RMSE (Root-Mean-Square Error) of 8.1317° and is generally between \(-13^\circ\) and \(11^\circ\), although larger errors occur around the jump from \(-90^\circ\) to \(90^\circ\) of estimated electrical angle. This increased error is mainly due to a shift of the estimated electrical angle of \(10^\circ\) compared to the case of \(\delta_s = 0.0\) which result in the jumps to occur at different mechanical angles. As this additional error of \(180^\circ\) does not affect the position estimation once a cosine function is applied, it is already removed in Figure 8 by adding \(180^\circ\). Nevertheless, such a significant deviation from the real electrical position may already be sufficient to reduce the efficiency of the electrical machine noticeably or even destabilize the controller driving the machine.

It is also worth mentioning that the non-zero mean value of \(\Gamma_{abc}\) in combination with the \(\arctan 2\) function of equation (7) results in a non-zero mean value of \(\chi\), in this case the mean value of \(\chi\) is \(-8.87^\circ\). This effect can already be observed in Figure 8 as the error is not symmetrical around zero. For very high SE with \(\delta_s > 0.6\) this effect amplifies to \(\chi\) not even accepting values between \(\pm90^\circ\) as shown in Figure 9. This makes driving the machine using this sensorless technique impossible. However, operating a machine with such a high degree of eccentricity is dangerous anyways and will result in other issues such as a highly fluctuating output torque as well.

In addition to the beforementioned shift, SE influence the harmonics of the position estimation significantly, as illustrated in Figure 10. This Figure shows the harmonic content of the deviation \(\Delta \chi\) shown in Figure 8 which are, once again, exclusively multiples of \(2p\). These deviations from the original angle \(\chi^*\) introduce steep sections in the graph of Figure 7 which make an unambiguous position
IV. EXPERIMENTAL RESULTS

In order to verify the results presented in the previous section, an experimental setup, as shown in Figure 11, capable of introducing specific characteristics of SE is implemented. Due to the increasingly high reluctance forces between the permanent magnets and the soft magnetic stator core for small air-gaps, high degrees of SE cannot be introduced correctly. The main reason is the limited mechanical stiffness of the setup which is insufficient for such high forces and allows the rotor and stator to skew for high values of δs. Thus, only results for δs < 0.5 are presented in this section.

The bearings of the used electrical machine, a ten pole twelve slot PMSM, are removed in order to facilitate free movement perpendicular to the rotation axis. An XYZ-stage is used to set a specific SE while a rotational stage rotates the rotor around the stator. The DFC signals of an entire mechanical period are then recorded and are presented in Figure 12 for two different SEs. The magnitude and angle of the SEs are relative to the estimated center position. Already in this figure a change of mean value as well as amplitude can be seen, further emphasized by the harmonic content presented in Figure 13. This reflects a similar behavior as predicted by the model and Figures 5 and 6 with the exception of also affecting harmonics which are not multiples of 2p. These harmonics likely exist due to minor errors and tolerances in the mechanical construction of the machine but are of considerably lesser amplitude compared to the 10th harmonic.

Applying equation (7) on these measurements leads to the estimated electrical angle and the results presented in Figure 14. Figure 15 instead shows the error between the position estimations, ranging from about −20° to 20° and a RMSE of 6.2514°. This angular deviation Δχ is similar to what the model predicted although the shift and resulting increased error in electrical rotor position at χ = ±90° is not apparent in the experimental results. Therefore, this effect will require additional investigation concerning both the analytical model and the measurements. Instead, the deviation is quite noisy and shows higher peak values which can be attributed to the additional harmonics seen in Figure 13. This observation is also reflected in the harmonic content of Δχ as presented in Figure 16. In this figure, some harmonics which are not multiples of 2p show a similar amplitude compared to those which are multiples of 2p. Nevertheless, these
auxiliary harmonics show a significant lower amplitude than the 10th and 20th harmonics. Nonetheless, the experimental results proof that, depending on the application, this error is quite significant and has to be considered for high accuracy positioning applications.

V. Conclusion

This work has presented and discussed the error that SE (Static Eccentricity) introduces to the position estimation when using the zero-sequence based sensorless technique DFC. This technique relies on the rotor position dependency of the phase inductances which vary due to SE. An analytical model capable of calculating this change in inductance was inserted into the mathematical framework derived by a sensitivity analysis of the DFC position estimation on variations in the inductance matrix. At the investigated SE of $\Delta_s = 0.4$, the calculated angular deviation is in the same range of about $\pm 13^\circ$ as the ones obtained experimentally, $\pm 20^\circ$, and was shown to be quite significant. The predicted shift of the estimated position is not present in the experimental data which instead exhibit numerous additional mechanical harmonics leading to an increased peak error. Therefore, both these effects require additional investigation.

References


